

# Cascading Failures in Power Grids - Analysis and Algorithms



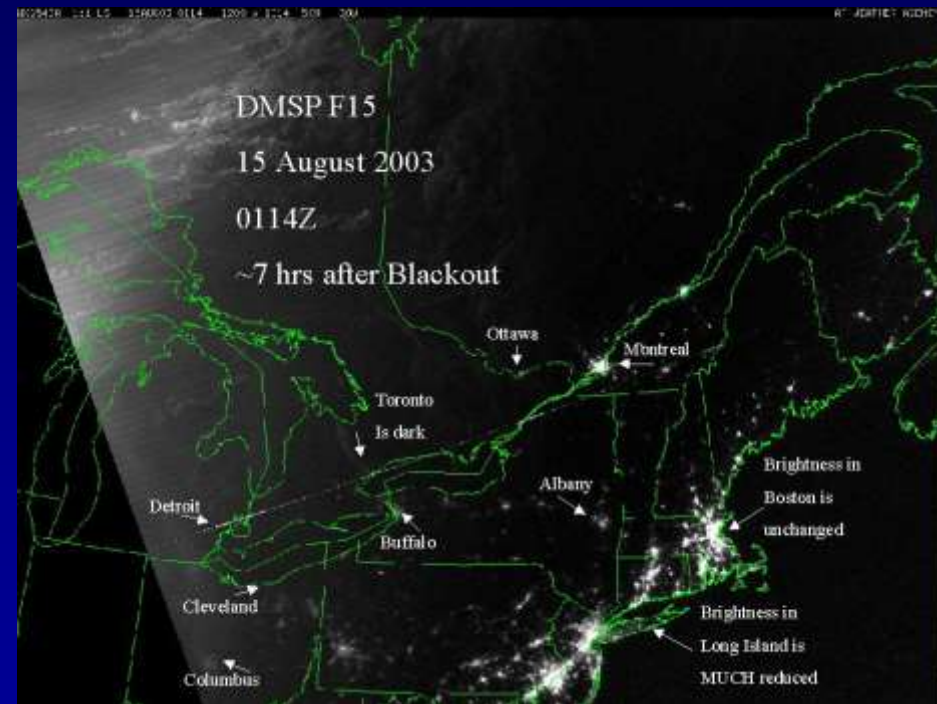
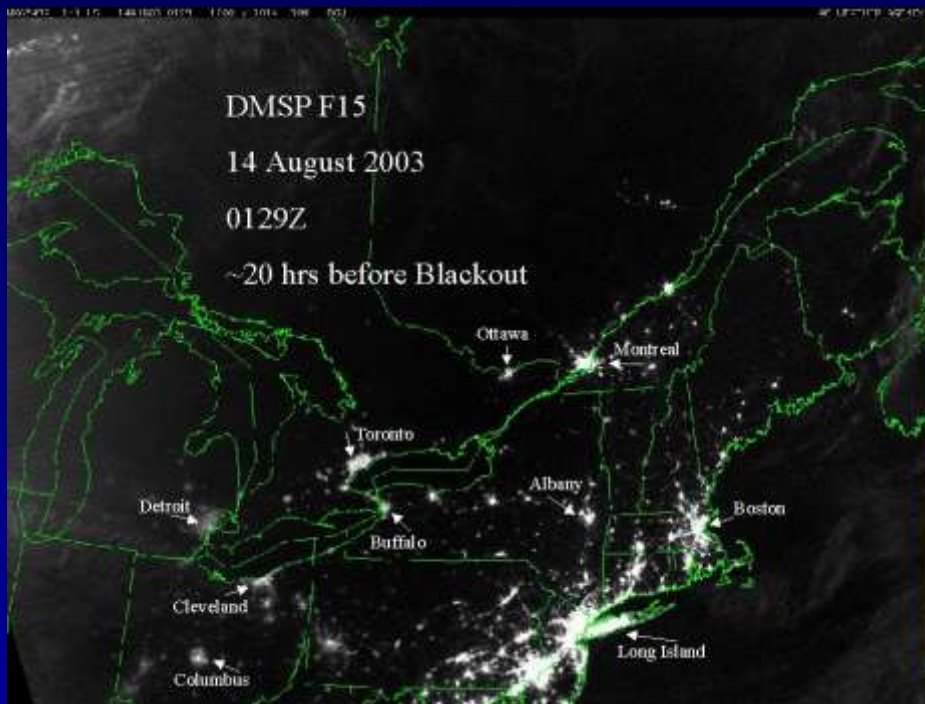
Saleh Soltan<sup>1</sup>, Dorian Mazaruic<sup>2</sup>, Gil Zussman<sup>1</sup>

<sup>1</sup> Electrical Engineering, Columbia University

<sup>2</sup> INRIA Sophia Antipolis

# Cascading Failures in Power Grids

- ◆ Power grids rely on physical infrastructure → Vulnerable to physical attacks/failures
- ◆ Failures may cascade



- ◆ An attack/failure will have a significant effect on many interdependent systems (communications, transportation, gas, water, etc.)

# Interdependent Networks

11/8/12

IEEE Communications Society | IEEE Communications Society

Connect:    

 IEEE  
COMMUNICATIONS  
SOCIETY

 IEEE

## Hurricane Sandy Update:

The effects of Hurricane Sandy are profound throughout the eastern seaboard of the United States, including the New York City metro area and vast portions of New

IEEE is experiencing significant po

tions in service from IEEE. We

## Hurricane Sandy Update

**IEEE is experiencing significant power disruptions to our U.S. facilities in New Jersey and New York. As a result, you may experience disruptions in service from IEEE.**

What's happening @C  
Find out at ComSoc B

[www.comsocblog.org](http://www.comsocblog.org)

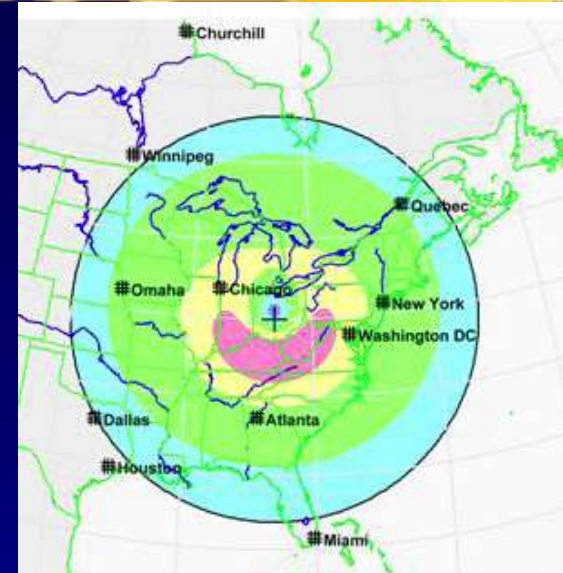
CALIFORNIA, USA +



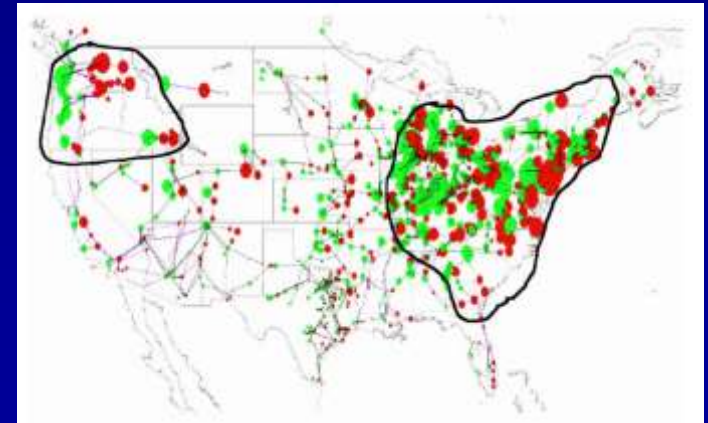
INIZING COMMITTEE

# Physical Attacks/Disasters

- ◆ EMP (Electromagnetic Pulse) attack
- ◆ Solar Flares - in 1989 the Hydro-Quebec system collapsed within 92 seconds leaving 6 Million customers without power



Source: Report of the Commission to Assess the threat to the United States from Electromagnetic Pulse (EMP) Attack, 2008



- ◆ Other natural disasters
- ◆ Physical attacks

FERC, DOE, and DHS, Detailed Technical Report on EMP and Severe Solar Flare Threats to the U.S. Power Grid, 2010

# Sniper Attack on a San Jose Substation, Apr. 2014



## Shots in the Dark

A look at the April 16 attack on PG&E's Metcalf Transmission Substation

| 1  | 2   | 3   | 4  | 5  | 6   | 7   |
|--|---|---|--|--|---|---|
| <b>12:58 a.m.,<br/>1:07 a.m.</b><br>Attackers cut<br>telephone<br>cables | <b>1:31 a.m.</b><br>Attackers<br>open fire on<br>substation | <b>1:41 a.m.</b><br>First 911 call<br>from power<br>plant<br>operator | <b>1:45 a.m.</b><br>Transformers<br>all over the<br>substation<br>start crashing | <b>1:50 a.m.</b><br>Attack ends<br>and gunmen<br>leave | <b>1:51 a.m.</b><br>Police arrive<br>but can't<br>enter the<br>locked<br>substation | <b>3:15 a.m.</b><br>Utility<br>electrician<br>arrives |

Sources: PG&E; Santa Clara County Sheriff's Dept.; California Independent System Operator; California Public Utilities Commission; Google (image)  
The Wall Street Journal

Source: Wall  
Street Journal

# Cascading Failures - Related Work

- ◆ Report of the Commission to Assess the threat to the United States from Electromagnetic Pulse (EMP) Attack, 2008
- ◆ Federal Energy Regulation Commission, Department of Energy, and Department of Homeland Security, Detailed Technical Report on EMP and Severe Solar Flare Threats to the U.S. Power Grid, Oct. 2010
- ◆ Cascading failures in the power grid
  - Dobson et al. (2001-2010), Hines et al. (2007-2010), Chassin and Posse (2005), Gao et al. (2011),...
  - The  $N-k$  problem where the objective is to find the  $k$  links whose failures will cause the maximum damage: Bienstock et al. (2005, 2009)
  - Interdiction problems: Bier et al. (2007), Salmeron et al. (2009), ...
  - Cascade control: Pfitzner et al. (2011), ...
  - Mostly do not consider computational aspects



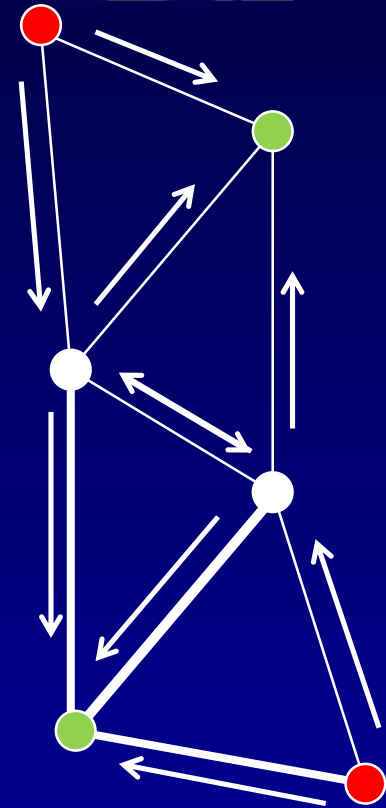
# Outline

---

- ◆ Background
- ◆ Power flows and cascading failures
  - Real events, models, and simulations
- ◆ Impact of single line failures
- ◆ Pseudo-inverse of the admittance matrix and resistance distance
- ◆ Efficient algorithm for cascade evolution
- ◆ Vulnerability analysis

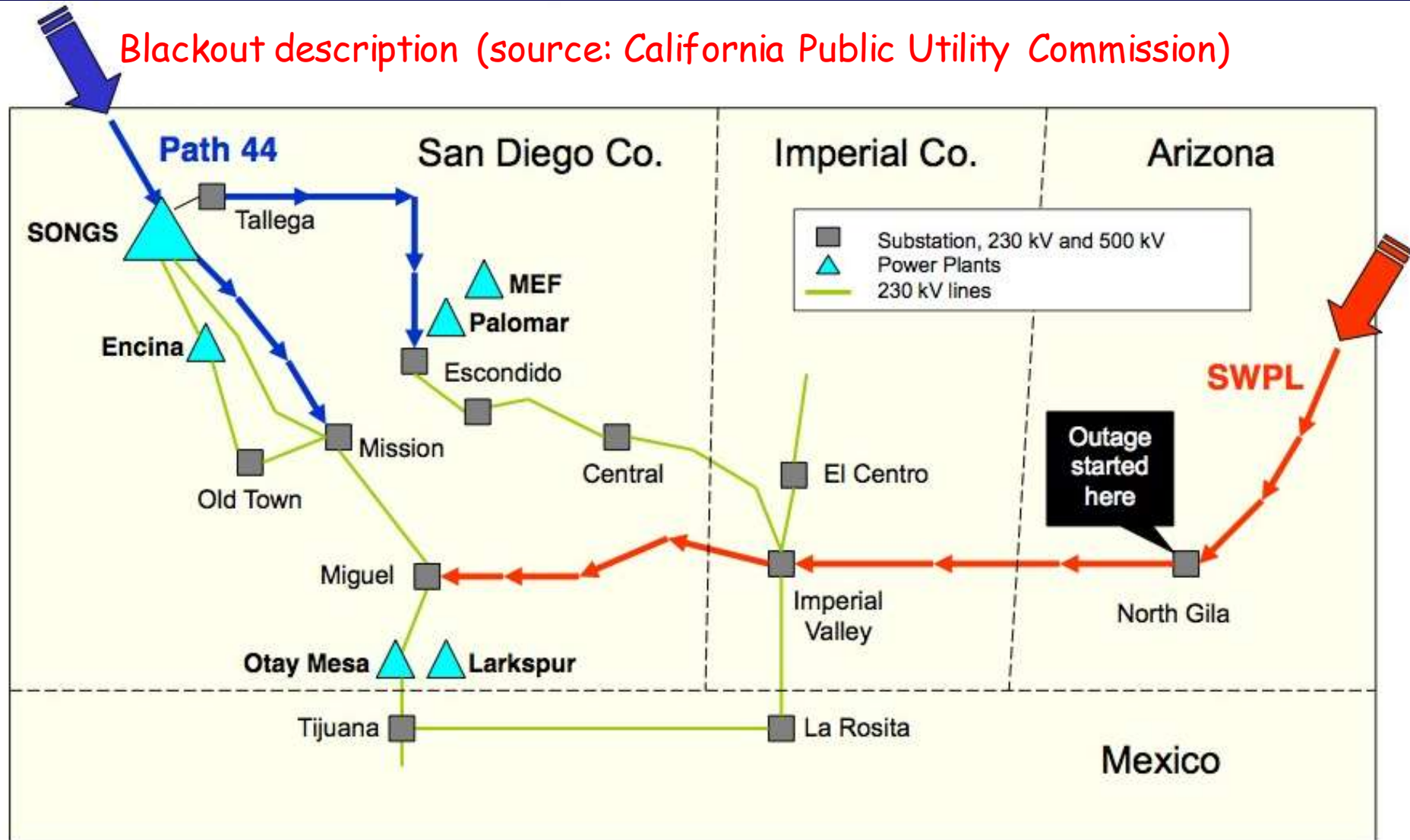
# Power Grid Vulnerability and Cascading Failures

- ◆ Power flow follows the laws of physics
- ◆ Control is difficult
  - It is difficult to “store packets” or “drop packets”
- ◆ Modeling is difficult
  - Final report of the 2003 blackout - cause #1 was “inadequate system understanding” (stated at least 20 times)
- ◆ Power grids are subject to **cascading failures**:
  - Initial failure event
  - Transmission lines fail due to overloads
  - Resulting in subsequent failures



# Recent Major Blackout Event: San Diego, Sept. 2011

Blackout description (source: California Public Utility Commission)



\*Map not to scale

# Event Timeline

Prior to start of events, SWPL delivering **1370 MW**, and Path 44 delivering **1287 MW**.

**15:27:39** – 500kV Hassayampa-North Gila (SWPL) line trips at North Gila Substation.

SWPL lost. Increased flow on Path 44 to **2407 MW**.

**15:27:58 to 15:30:00** – CCM tripped in CFE area (needed emergency assistance of 158 MW). IID experienced problems with Imperial Valley-EI Centro line resulting in 100MW swing.

Path 44 flow increased to **2616 MW**.

**15:32:00 to 15:33:44** – IID transformer bank and two units trip. Also two 161 kV lines trip at Niland-WAPA and Niland-Coachella Valley.

Flow from SDG&E to IID increased by 209 MW. Path 44 flow increased to **2959 MW**.

**15:35:40 to 15:36:45** – Two APS 161 kV lines to Yuma tripped and electrically separated from IID and WAPA. SDG&E now fed power into Yuma area.

**15:37:56** – IID's 161 kV tie to WAPA tripped. Import power into Yuma, Imperial Valley, Baja Norte, and San Diego wholly dependant on Path 44.

Flow from SONGS to San Diego to Yuma. Path 44 flow increased to **3006 MW**.

**15:37:58 to 15:38:07** – EI Centro Substation (IID) trip due to under frequency. Two units at La Rosita plant (CFE) trip resulting in a loss of 420 MW.

Path 44 flow increased to **3454 MW and 7500 Amps**.

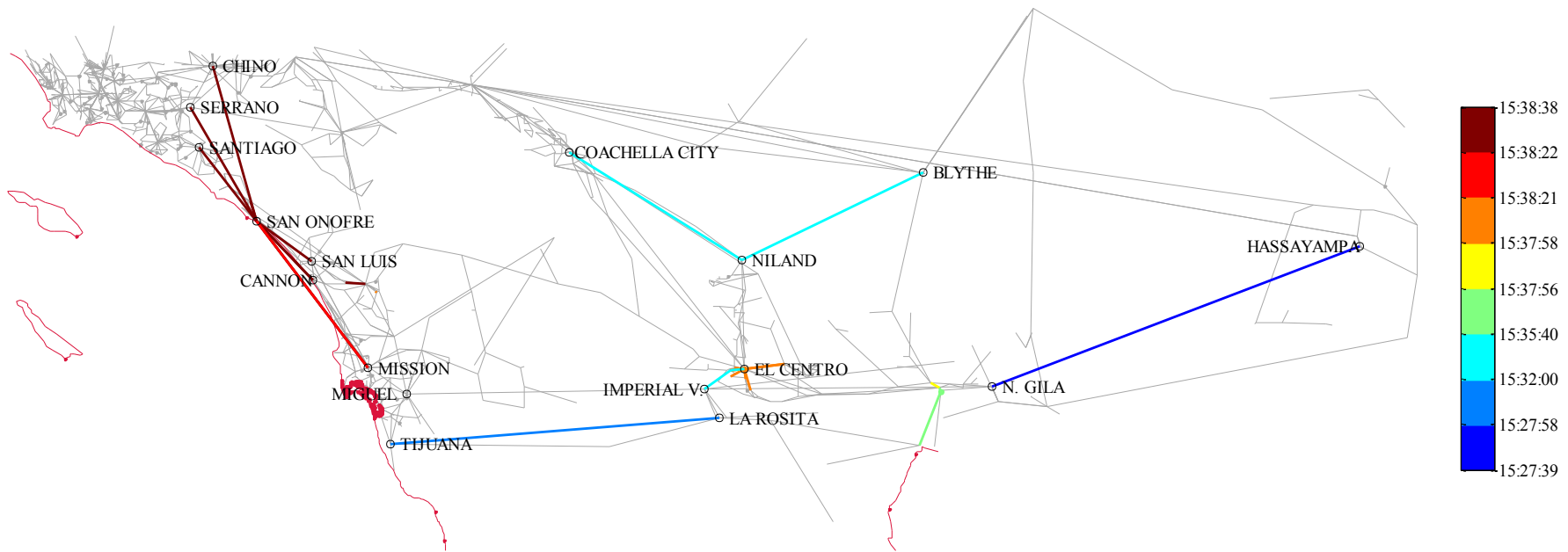
**15:38:21** – Path 44 exceeded safety setting of 8000 Amps. Overload relay protection initiated to separate Path 44 between SCE and SDG&E at SONGS switchyard.

**15:38:22 to 15:38:38** – SONGS and local power plants trip. 230kV lines open.

Path 44 reaches **9660 Amps**, then drops to **8230 Amps**.

**15:38:38** – Blackout

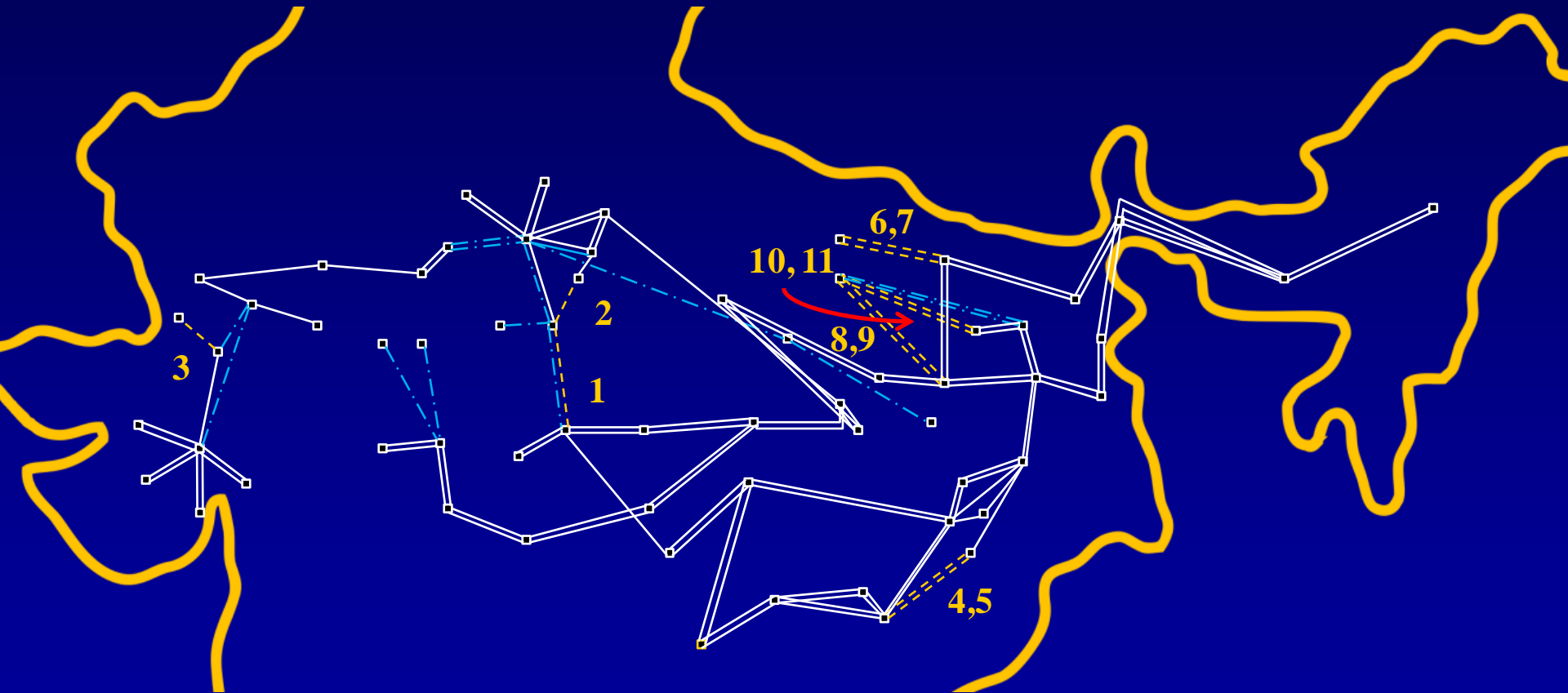
# Real Cascade - San Diego Blackout



- ◆ Failures "skip" over a few hops
- ◆ Does not agree with the epidemic/percolation models

# Blackout in India, July 2012

- ◆ The first 11 line outages leading to the India blackout on July 2012 (numbers show the order of outages)



—— Functional    - - - - Under Maintenance    - - - - Tripped

# Power Flow Equations - DC Approximation


- Exact solution to the AC model is infeasible

$$f_{ij} = U_i^2 g_{ij} - U_i U_j g_{ij} \cos \theta_{ij} - U_i U_j b_{ij} \sin \theta_{ij}$$

$$Q_{ij} = -U_i^2 b_{ij} + U_i U_j b_{ij} \cos \theta_{ij} - U_i U_j g_{ij} \sin \theta_{ij}$$

$$\text{and } \theta_{ij} = \theta_i - \theta_j.$$

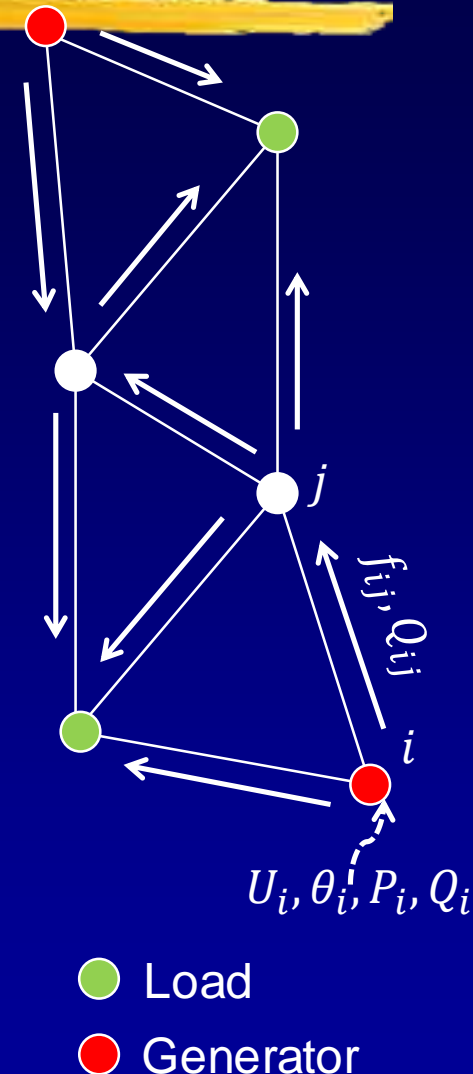
- We use **DC approximation** which is based on:

$$U_i \equiv 1, \forall i$$


$$x_{ij}$$

$$\sin \theta_{ij} \approx \theta_{ij}$$

- $U_i = 1$  p.u. for all  $i$
- Pure reactive** transmission lines - each line is characterized only by its reactance  $x_{ij} = -1/b_{ij}$
- Phase angle differences are "small", implying that  $\sin \theta_{ij} \approx \theta_{ij}$
- Known as a reasonably good approximation
- Frequently used for contingency analysis
  - Do the assumptions hold during a cascade?



# Power Flow Equations - DC Approximation

- ◆ A power flow is a solution  $(f, \theta)$  of:

$$\sum_{v \in N(u)} f_{uv} = p_u, \forall u \in V$$

$$\theta_u - \theta_v - x_{uv} f_{uv} = 0, \forall \{u, v\} \in E$$

- ◆ Matrix form:

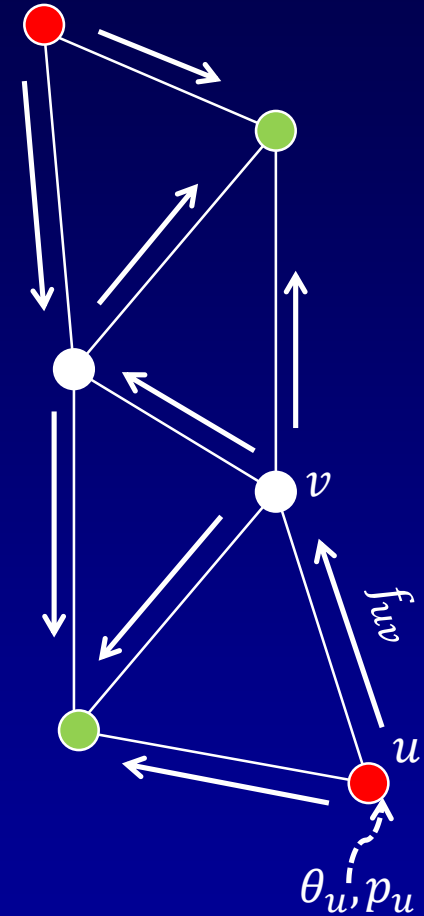
$$A\Theta = P$$

$A$  is the admittance matrix of the grid defined as:

$$a_{uv} = \begin{cases} 0, & u \neq v \text{ and } \{u, v\} \notin E \\ -\frac{1}{x_{uv}}, & u \neq v \text{ and } \{u, v\} \in E \\ -\sum_{w \in N(u)} a_{vw}, & u = v \end{cases}$$

- ◆ If  $A^+$  is its *pseudo-inverse*

$$\Theta = A^+ P$$



- Load ( $p_u < 0$ )
- Generator ( $p_u > 0$ )

# Line Outage Rule

- ◆ Different factors can be considered in modeling outage rules
  - The main is **thermal capacity**  $u_{ij}$

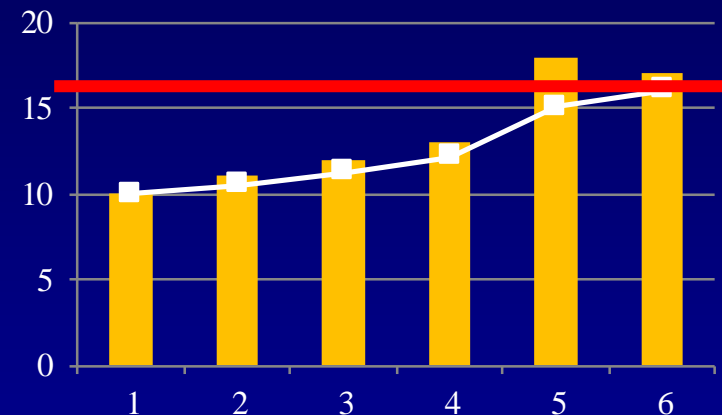
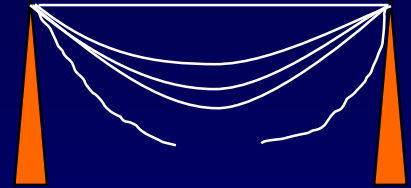
- ◆ Simplistic approach: fail lines with  $|f_{ij}| > u_{ij}$

*Not part of the power flow problem constraints*

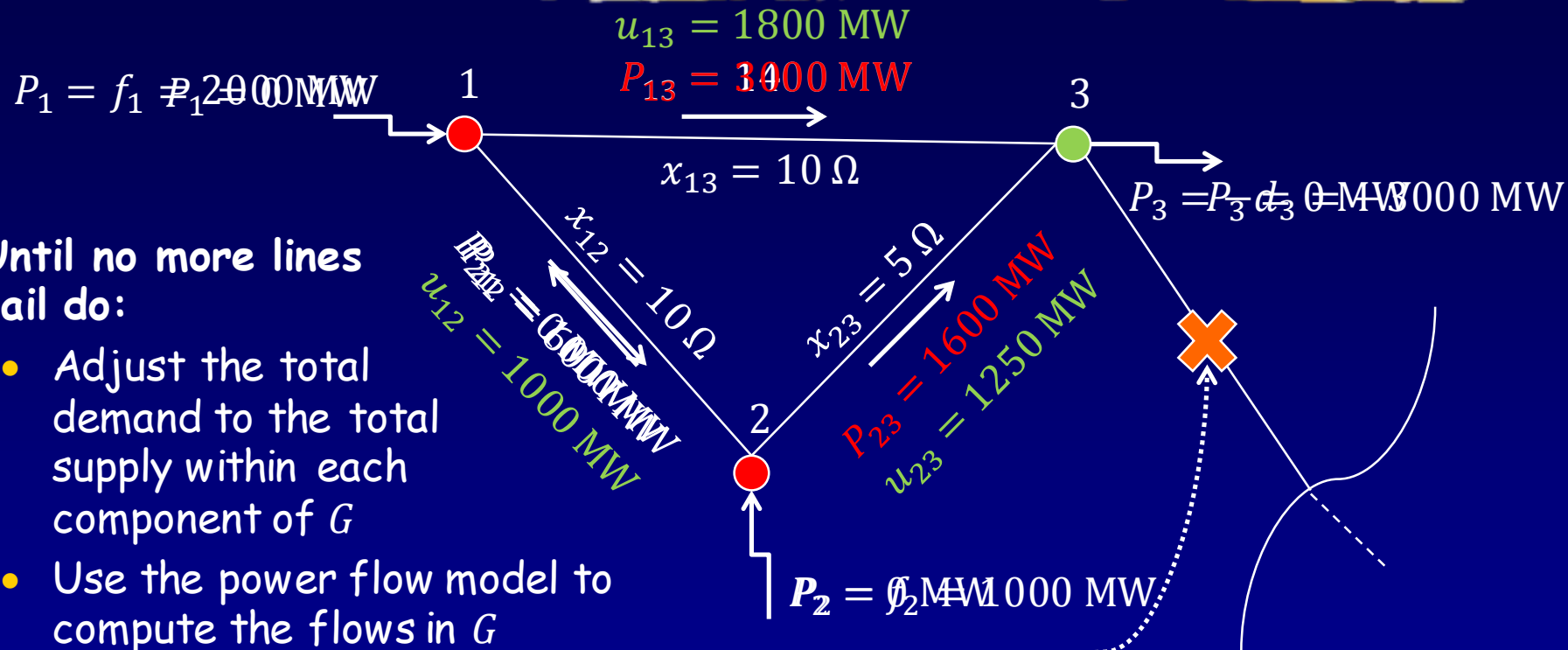
- ◆ More realistic policy:  
Compute the moving average  
 $\tilde{f}_{ij} := \alpha |f_{ij}| + (1 - \alpha) \tilde{f}_{ij}$   
( $0 \leq \alpha \leq 1$  is a parameter)

- ◆ **Deterministic outage rule:**  
Fail lines with  $\tilde{f}_{ij} > u_{ij}$

- ◆ **Stochastic outage rules**



# Cascading Failure Model (Dobson et al.)



## Until no more lines fail do:

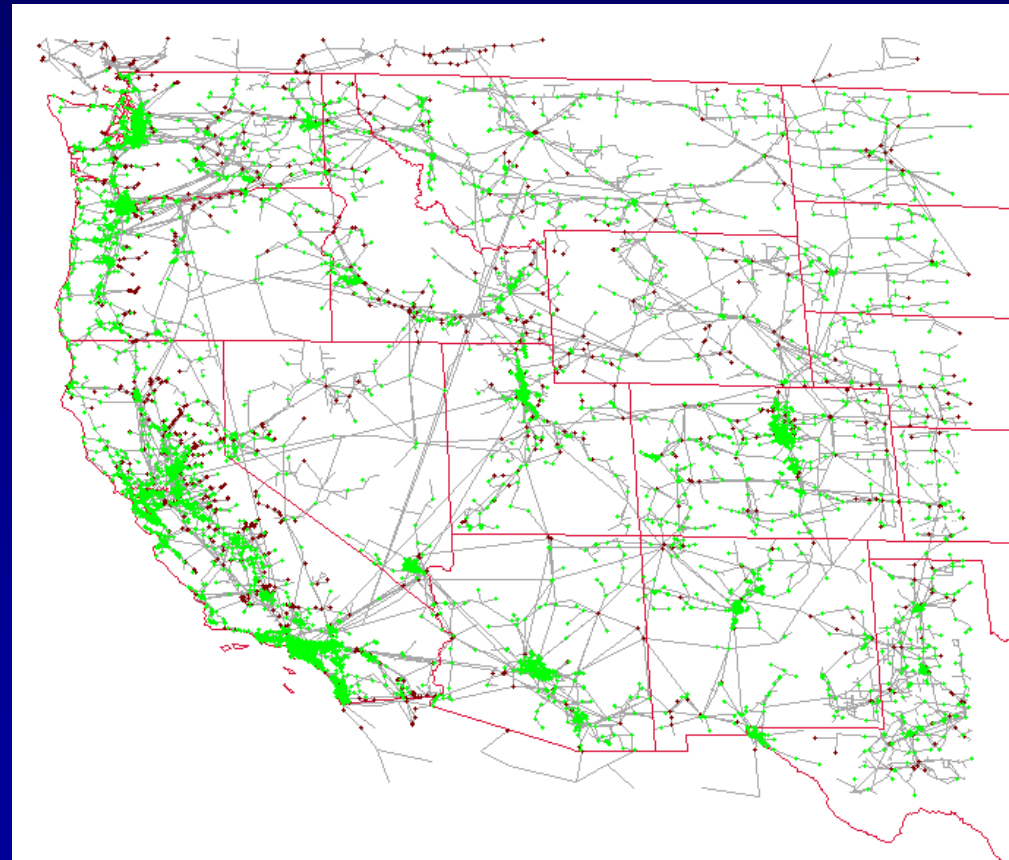
- Adjust the total demand to the total supply within each component of  $G$
- Use the power flow model to compute the flows in  $G$
- Update the state of lines  $\xi_{ij}$  according to the new flows
- Remove the lines from  $G$  according to a given outage rule  $O$

Initial failure causes disconnection of load 3 from the generators in the rest of the network

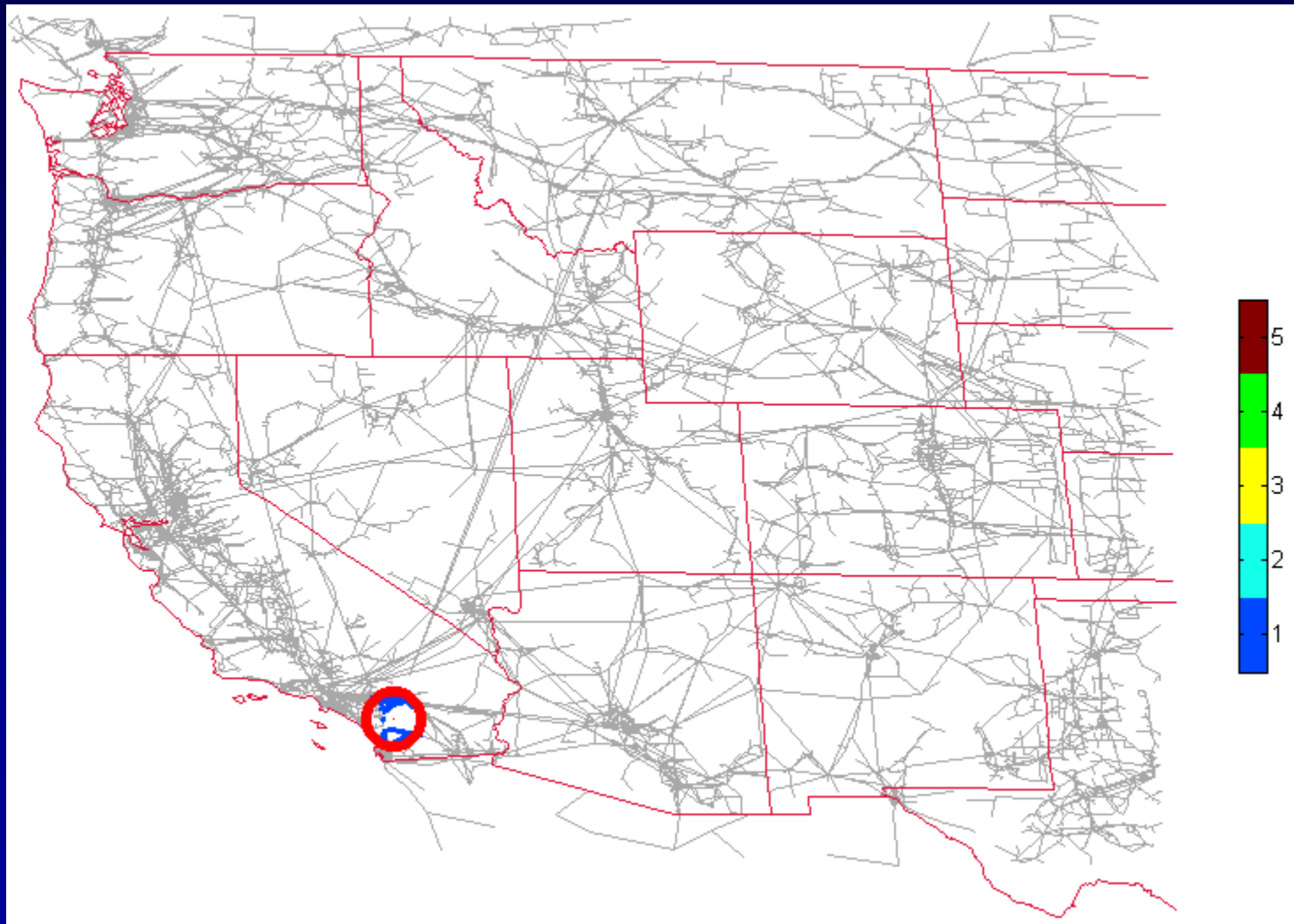
As a result, line (2,3) becomes overloaded

# Numerical Results (Bernstein et al., IEEE INFOCOM'14)

- ◆ Obtained from the GIS (Platts Geographic Information System)
- ◆ Substantial processing of the raw data
- ◆ Used a modified Western Interconnect system, to avoid exposing the vulnerability of the real grid
- ◆ 13,992 nodes (substations), 18,681 lines, and 1,920 power stations.
- ◆ 1,117 generators (red), 5,591 loads (green)
- ◆ Assumed that demand is proportional to the population size

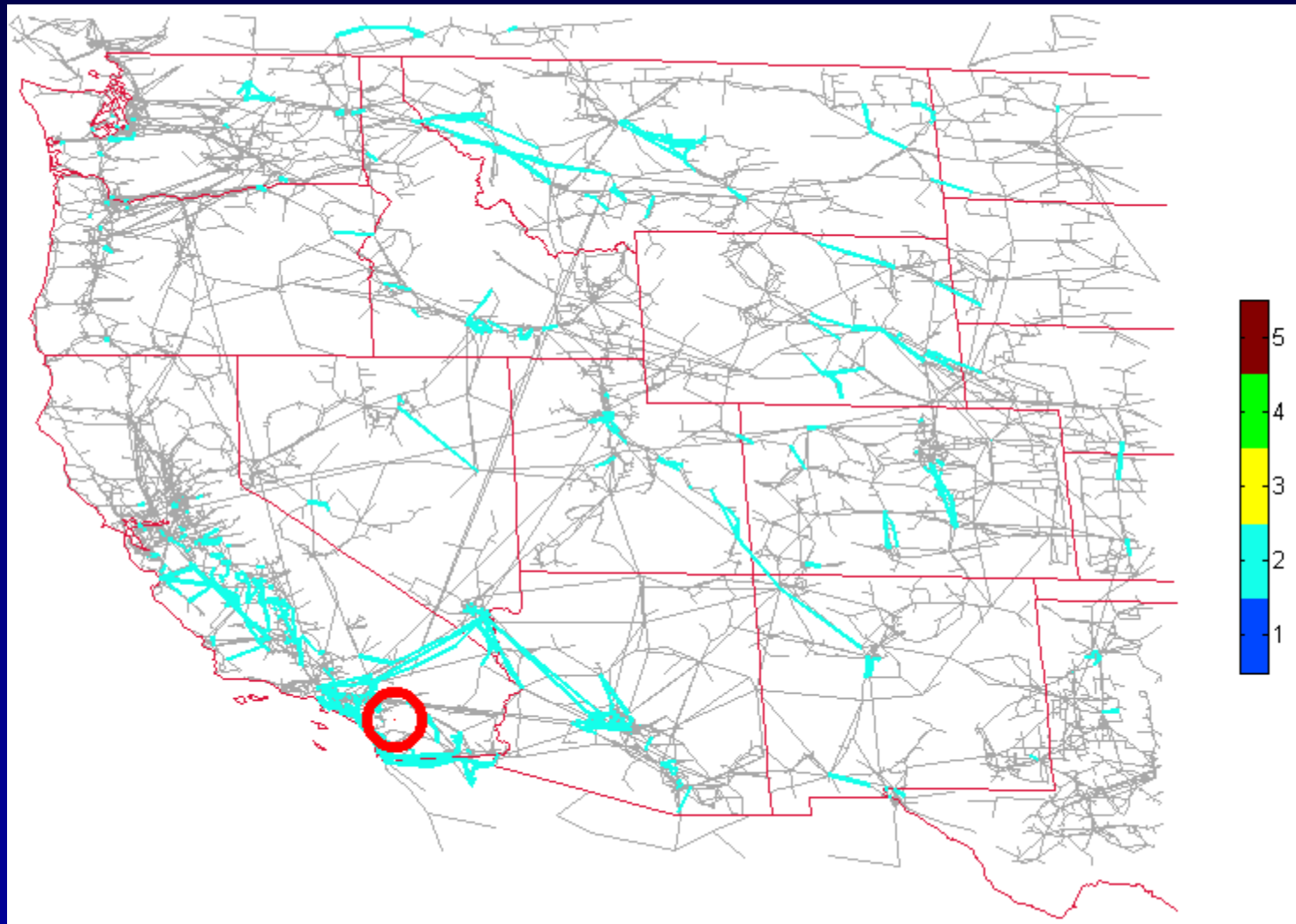


# Cascade Development - San Diego area

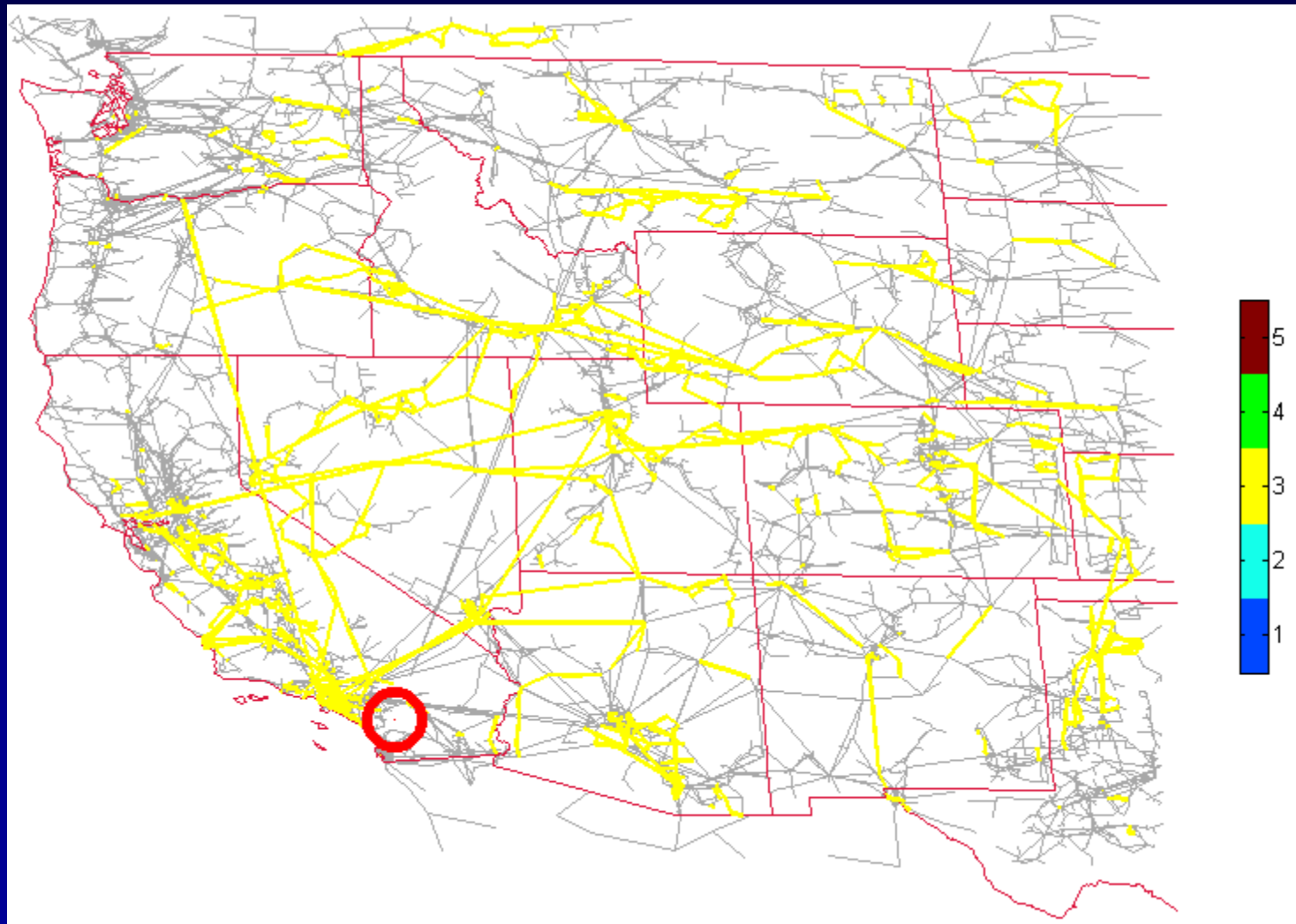


N-Resilient, Factor of Safety  $K = 1.2$

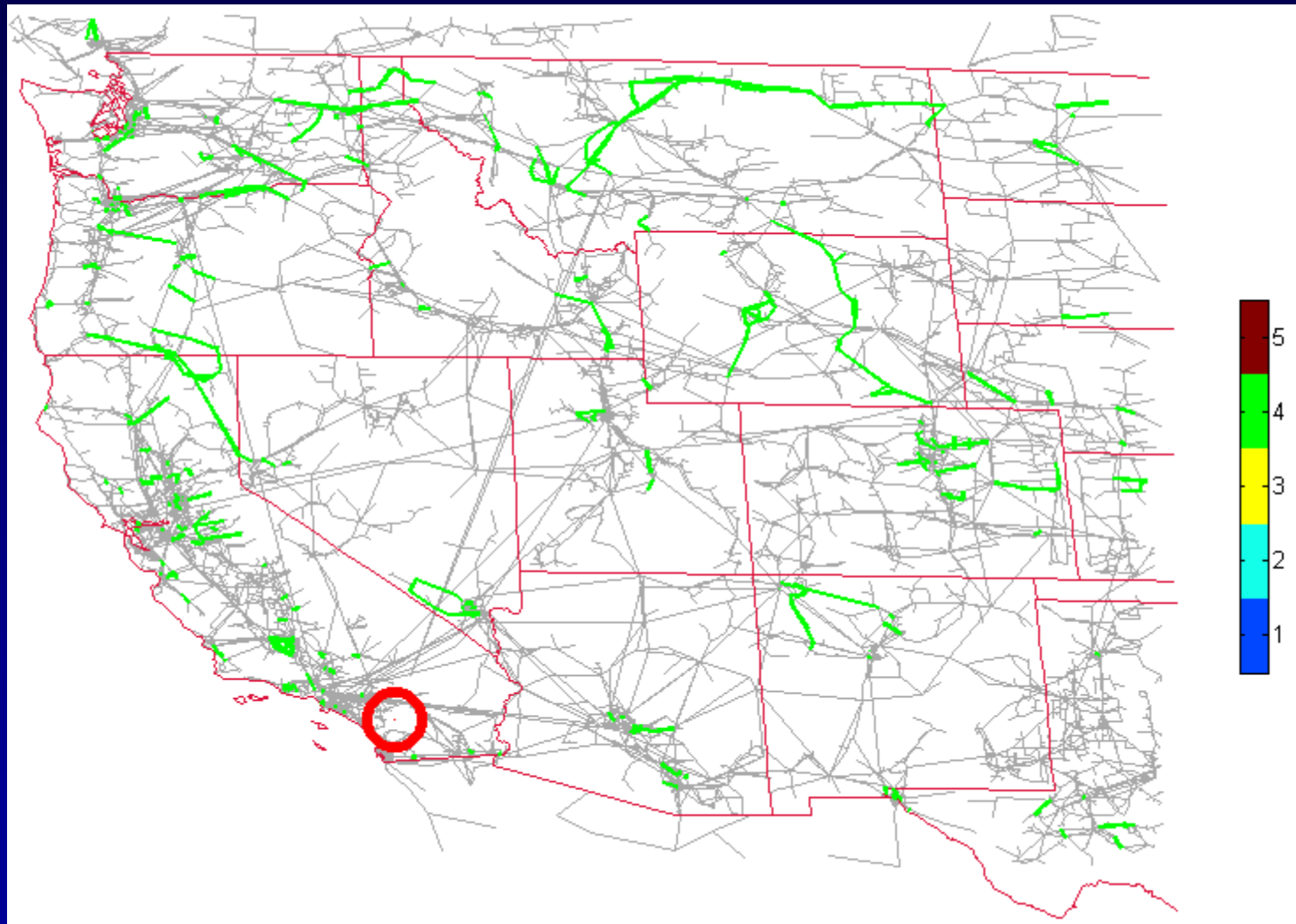
# Cascade Development - San Diego area



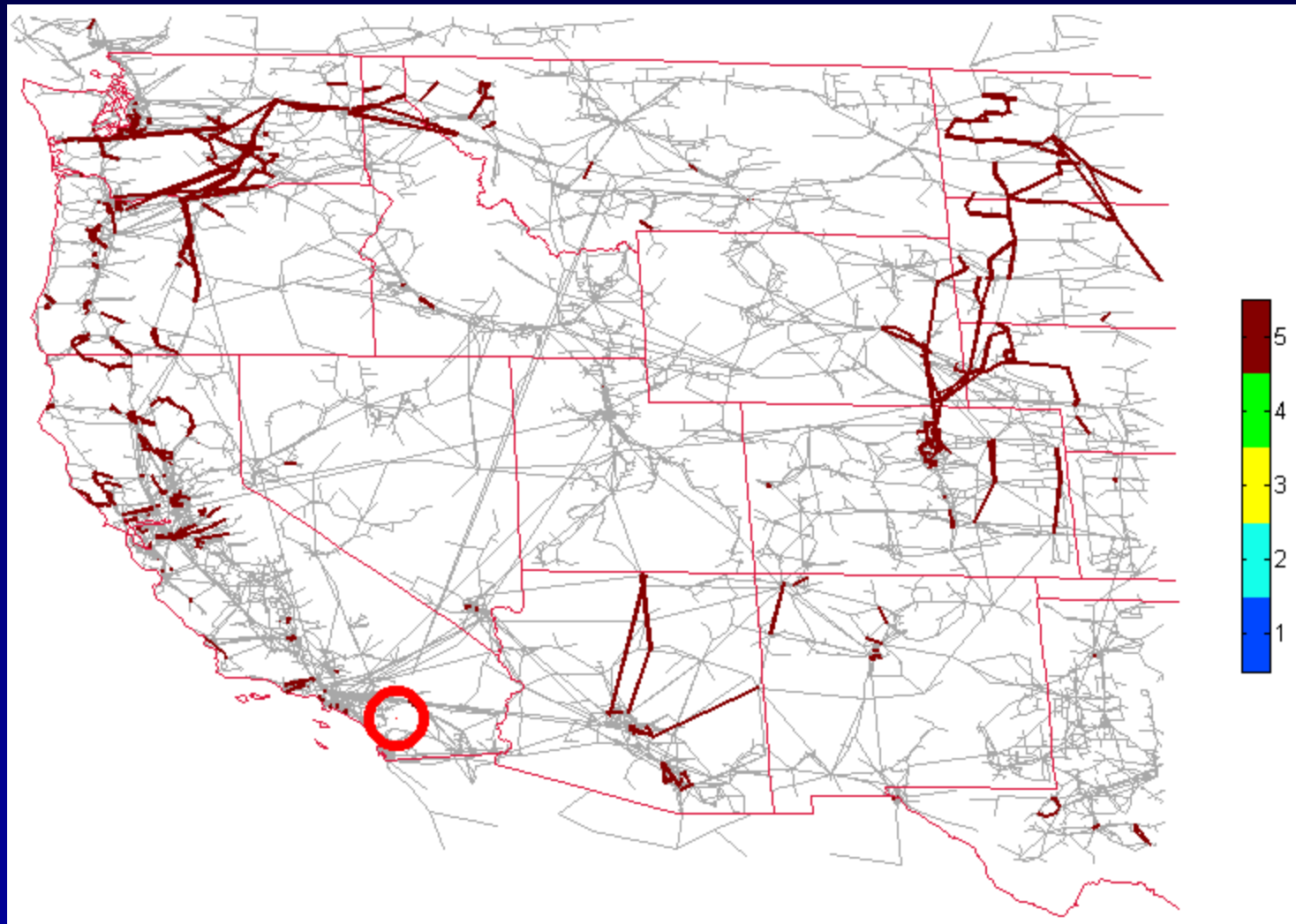
# Cascade Development - San Diego area



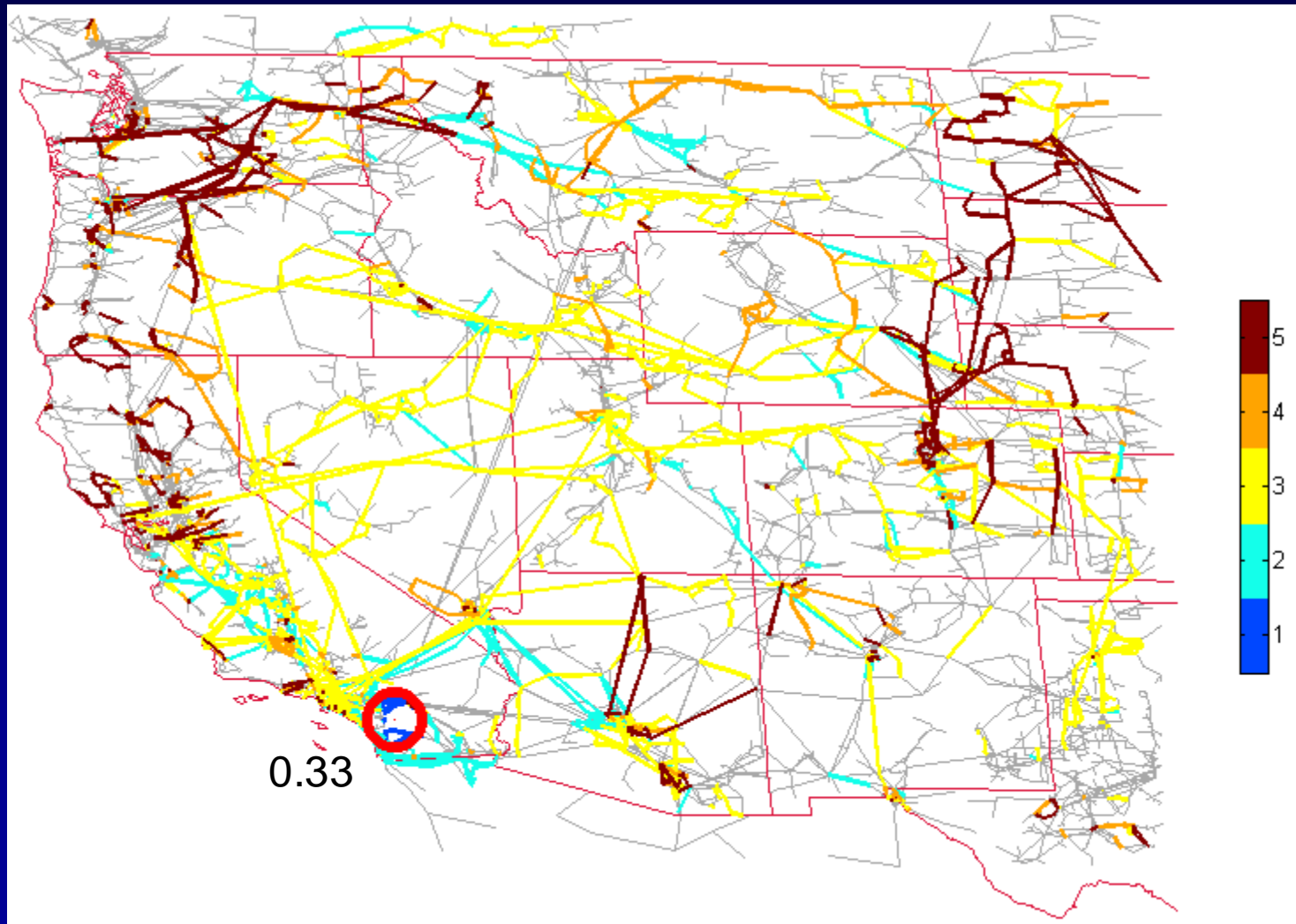
# Cascade Development - San Diego area



# Cascade Development - San Diego area



# Cascade Development - San Diego area



$N$ -Resilient, Factor of Safety  $K = 1.2 \rightarrow \text{Yield} = 0.33$

For  $(N-1)$ -Resilient  $\rightarrow \text{Yield} = 0.35$

For  $K = 2 \rightarrow \text{Yield} = 0.7$

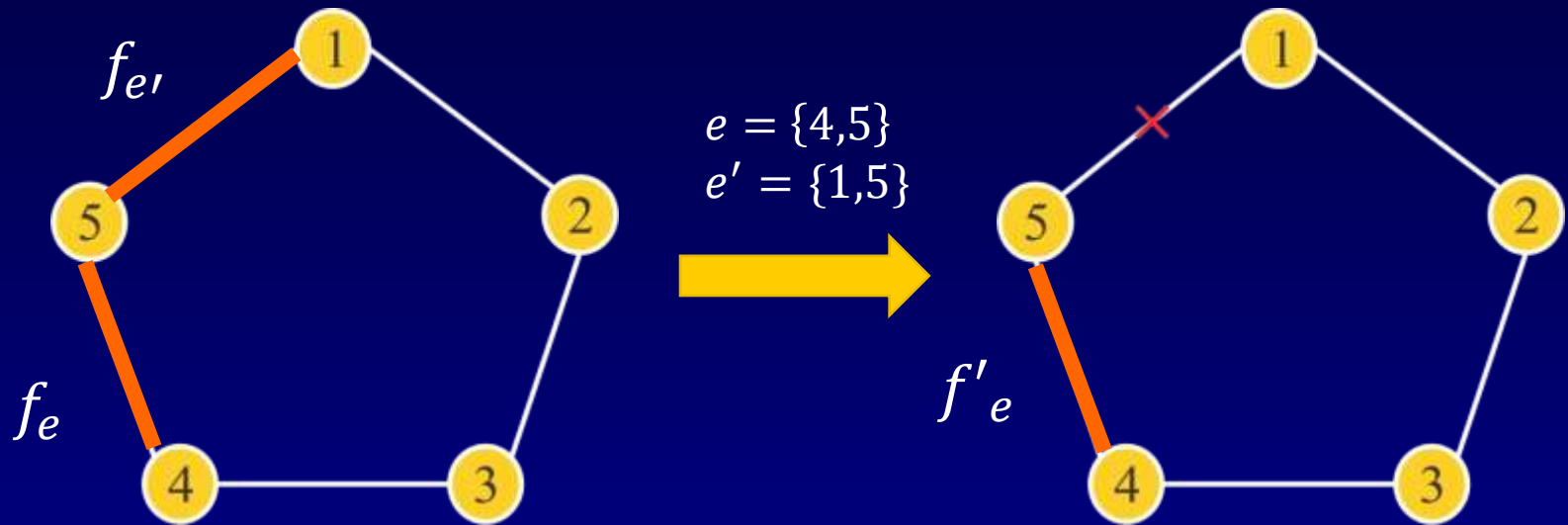
(Yield - the fraction of the demand which is satisfied at the end of the cascade)

# Outline

---

- ◆ Background
- ◆ Power flows and cascading failures
  - Real events, models, and simulations
- ◆ Impact of single line failures
- ◆ Pseudo-inverse of the admittance matrix and resistance distance
- ◆ Efficient algorithm for cascade evolution
- ◆ Vulnerability analysis

# Metrics for Evaluating the Impact of a Single Failure



- ◆ Flow Change after failure in the edge  $e$

$$\Delta f_e = f'_e - f_e$$

- ◆ Edge Flow Change Ratio

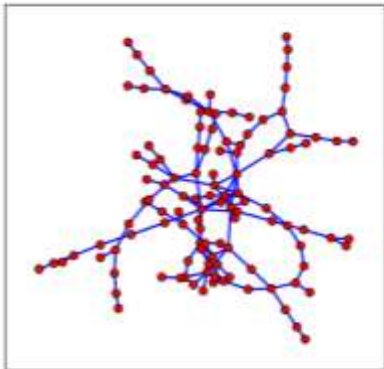
$$S_{e,e'} = \left| \frac{\Delta f_e}{f_e} \right|$$

- ◆ Mutual Edge Flow Change Ratio

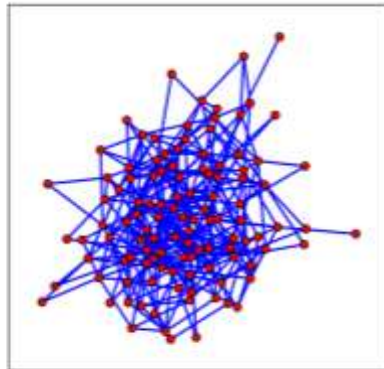
$$M_{e,e'} = \left| \frac{\Delta f_e}{f_{e'}} \right|$$

# Graph Used in Simulations

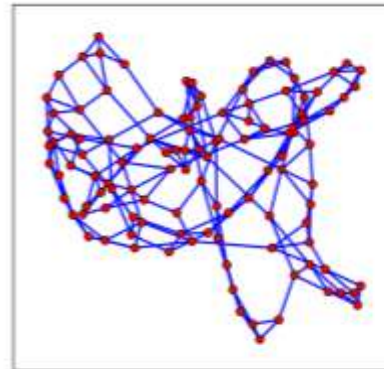
- ◆ **Western interconnection:** 1708-edge connected subgraph of the U.S. Western interconnection
- ◆ **Erdos-Renyi graph:** A random graph where each edge appears with probability  $p = 0.01$
- ◆ **Watts and Strogatz graph:** A small-world random graph where each node connects to  $k = 4$  other nodes and the probability of rewiring is  $p = 0.1$
- ◆ **Barabasi and Albert graph:** A scale-free random graph where each new node connects to  $k = 3$  other nodes at each step following the preferential attachment mechanism



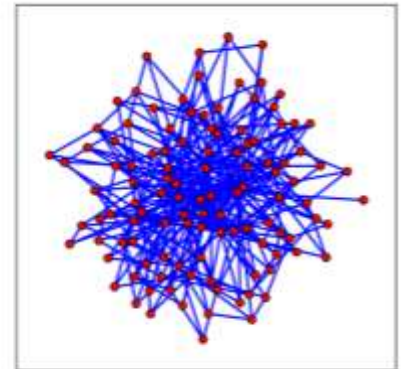
(a) Western interconnection



(b) Erdos-Renyi



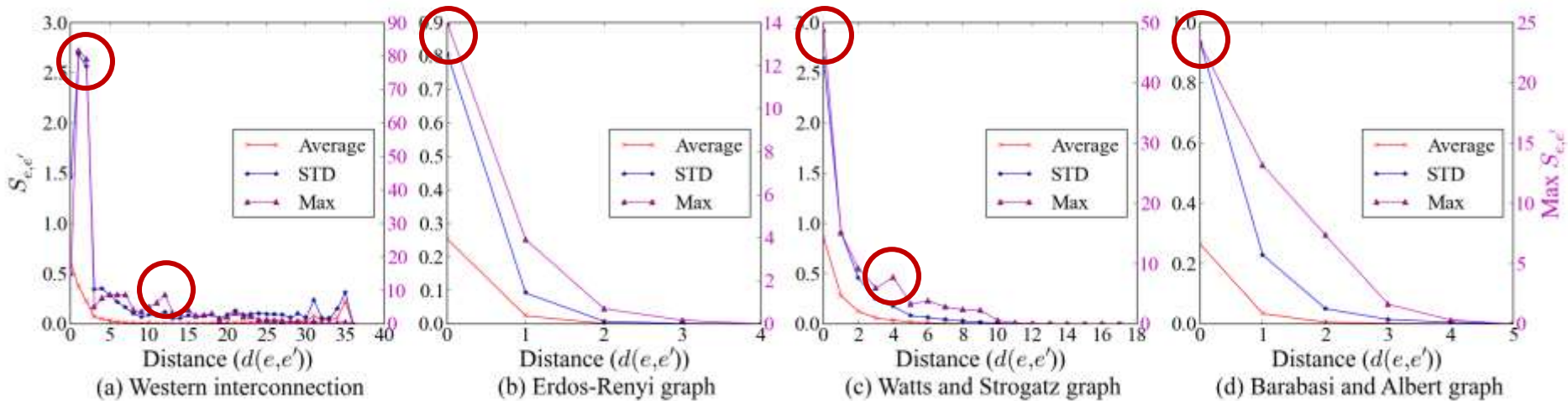
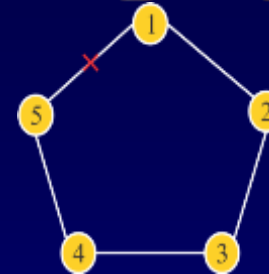
(c) Watts and Strogatz



(d) Barabasi and Albert

# Effects of a Single Edge Failure

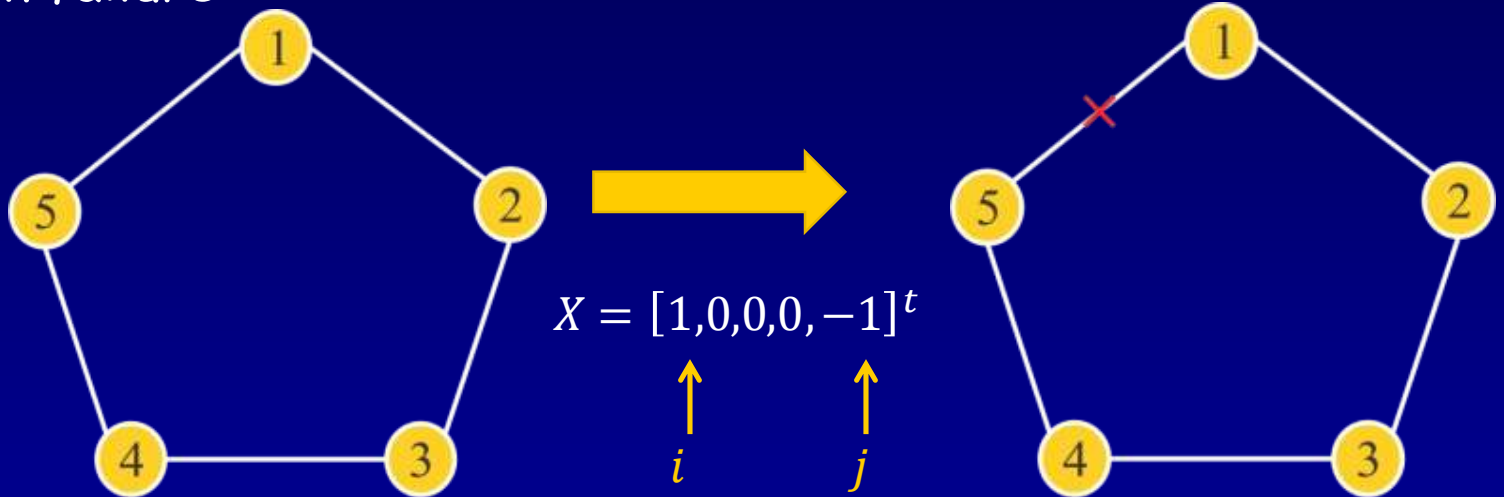
- ◆ Edge Flow Change Ratio,  $S_{e,e'} = \left| \frac{\Delta f_e}{f_e} \right|$



- ◆ Very large changes in the flow
- ◆ Sometimes far from the failure
- ◆ There are edges with positive flow increase from zero, far from the initial edge failure

# Updating the Pseudo-Inverse

- ◆ **Objective:** Compute the mutual edge flow change ratios
- ◆ Recall that  $\Theta = A^+P$
- ◆ **Method:** Update the **pseudo-inverse** of the admittance matrix upon failure



Admittance Matrix:  $A$

Admittance Matrix:  $A'$

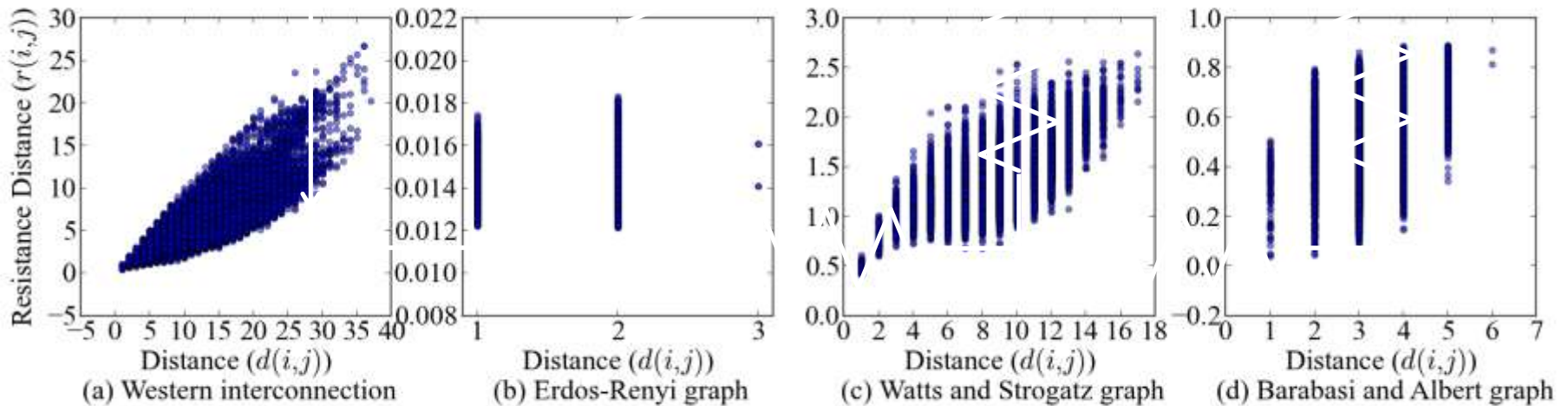
$$A' = (A + a_{ij}XX^t)$$

**Theorem:**  $A'^+ = (A + a_{ij}XX^t)^+ = A^+ - \frac{1}{a_{ij}^{-1} + X^t A^+ X} A^+ X X^t A^+$

\*A similar theorem independently proved by Ranjan et al., 2014

# Resistance Distance

- ◆ Resistance Distance between nodes  $i$  and  $j$   
$$r(i, j) = a_{ii}^+ + a_{jj}^+ - 2a_{ij}^+$$
- ◆ The resistance distance between two nodes is a measure of their connectivity



- ◆ All graphs have 1374 nodes
- ◆ All the edges have reactances equal to 1

# Effects of a Single Edge Failure

- ◆ Mutual Edge Flow Change Ratio

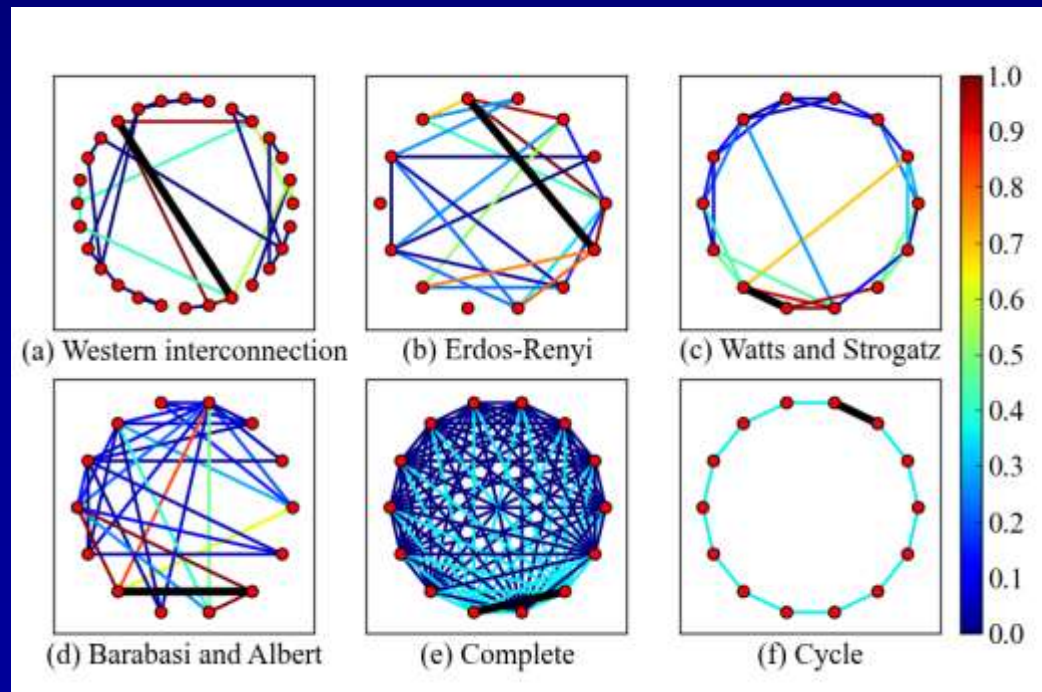
$$M_{e,e'} = \left| \frac{\Delta f_e}{f_{e'}} \right|$$

- ◆ Using pseudo-inverse of the admittance matrix,  $e = \{i, j\}$ ,  $e' = \{p, q\}$

$$M_{e,e'} = \frac{1}{2} \frac{1 - r(i, p) + r(i, q) + r(j, p) - r(j, q)}{1 - r(p, q)}$$

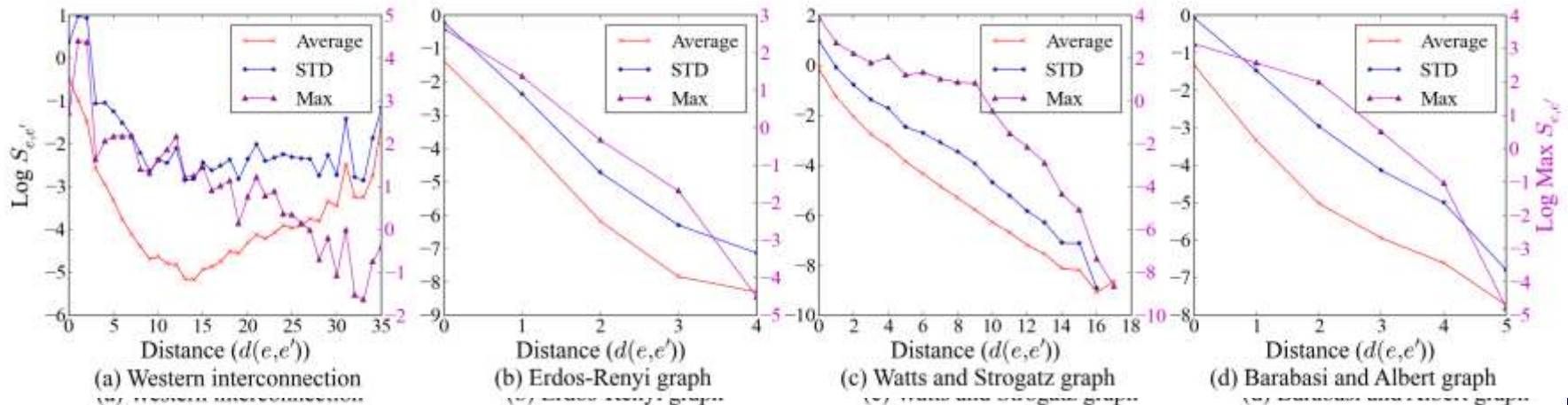
- ◆ Mutual Edge flow change ratios ( $M_{e,e'}$ ) are independent of the supply and demand


- ◆ Initial Failure  
Represented by a black wide line

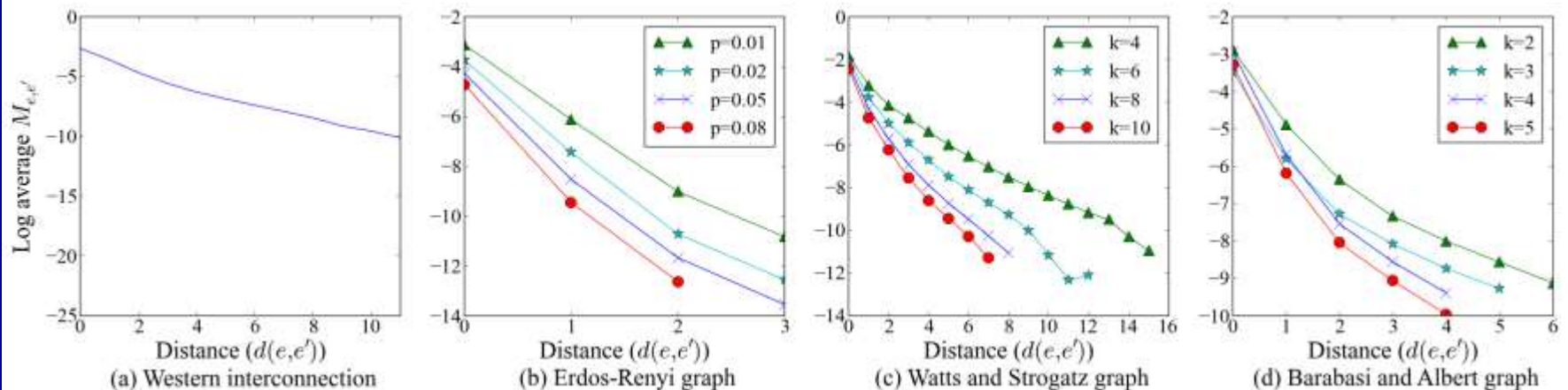


# Effects of a Single Edge Failure ( $M_{e,e'}$ )

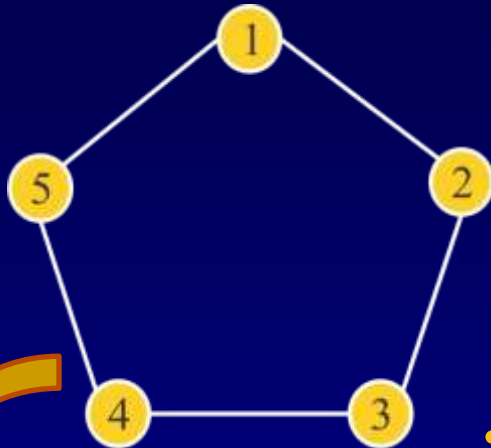
- ◆ All graphs have 1374 nodes and each point represents the average of 40 different initial single edge failure events



Logarithmic  Comparison



# Efficient Cascading Failure Evolution Computation

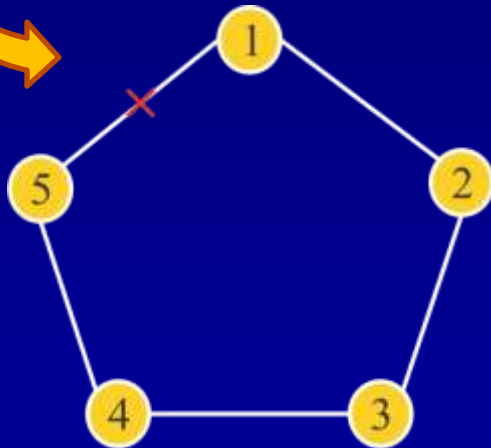


$$A = \begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ -1 & 0 & 0 & -1 & 2 \end{bmatrix}$$

$$A^+ = \begin{bmatrix} 0.4 & 0 & -0.2 & -0.2 & 0 \\ 0 & 0.4 & 0 & -0.2 & -0.2 \\ -0.2 & 0 & 0.4 & 0 & -0.2 \\ -0.2 & -0.2 & 0 & 0.4 & 0 \\ 0 & -0.2 & -0.2 & 0 & 0.4 \end{bmatrix}$$

- *Case I: Failure of an edge which is not a cut-edge*

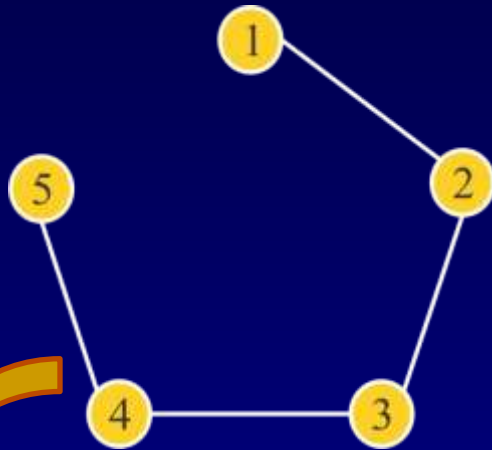
✓ Update  $A^+$  after removing the edge  $\{1,5\}$ , in  $O(|V|^2)$



$$A' = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

$$A'^+ = \begin{bmatrix} 1.2 & 0.4 & -0.2 & -0.6 & -0.8 \\ 0.4 & 0.6 & 0 & -0.4 & -0.6 \\ -0.2 & 0 & 0.4 & 0 & -0.2 \\ -0.6 & -0.4 & 0 & 0.6 & 0.4 \\ -0.8 & -0.6 & -0.2 & 0.4 & .2 \end{bmatrix}$$

# Efficient Cascading Failure Evolution Computation



$$A' = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

$$A'^+ = \begin{bmatrix} 1.2 & 0.4 & -0.2 & -0.6 & -0.8 \\ 0.4 & 0.6 & 0 & -0.4 & -0.6 \\ -0.2 & 0 & 0.4 & 0 & -0.2 \\ -0.6 & -0.4 & 0 & 0.6 & 0.4 \\ -0.8 & -0.6 & -0.2 & 0.4 & .2 \end{bmatrix}$$

- *Case II: Failure of a cut-edge*

✓ Detect the cut-edge and connected components in  $O(|V|)$

- ✓ Adjust the total demand to equal the total supply within each connected component
- ✓ No need to update  $A^+$

$$a'_{23}^{-1} - 2a'_{23}^{++} + a'_{22}^{++} + a'_{33}^{++} = 0$$

$$A'' = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

$$A_2'^+ - A_3'^+ = \begin{bmatrix} 0.6 & 0.6 & 0 & -0.4 & -0.4 \end{bmatrix}$$

$G_2$

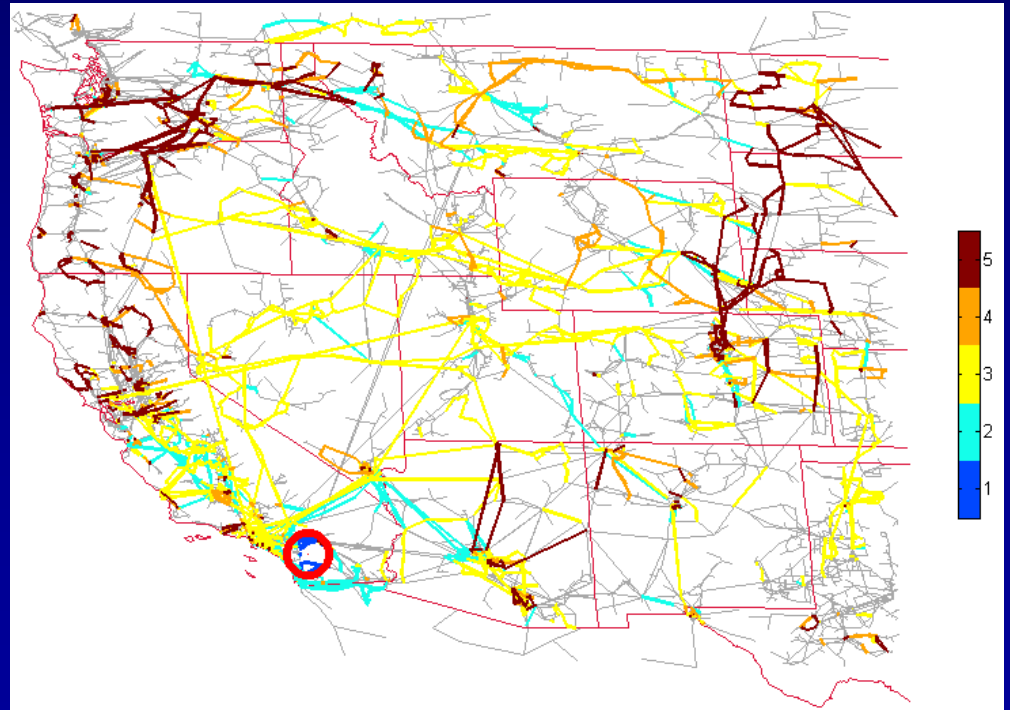
$G_1$

$G_1$

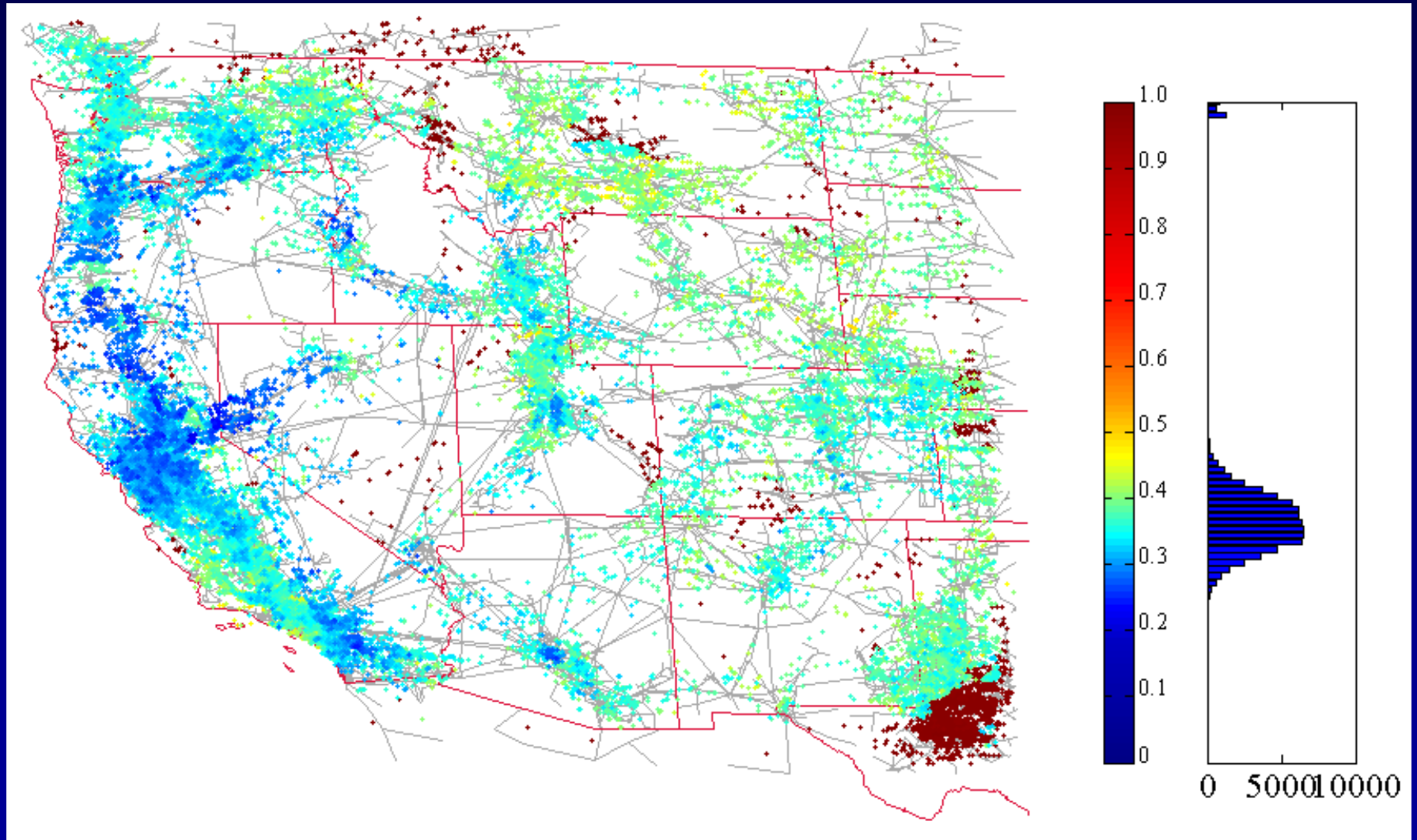
$G_2$

# Efficient Cascading Failure Evolution Computation

- ◆ The Pseudo-inverse Based Algorithm identifies the evolution of the cascade in  $O(|V|^3 + |F_t^*||V|^2)$
- ◆ Compared to the classical algorithm which runs in  $O(t|V|^3)$
- ◆ If  $t = |F_t^*|$  (one edge fails at each round), our algorithm performs  $O(\min\{|V|, t\})$  faster



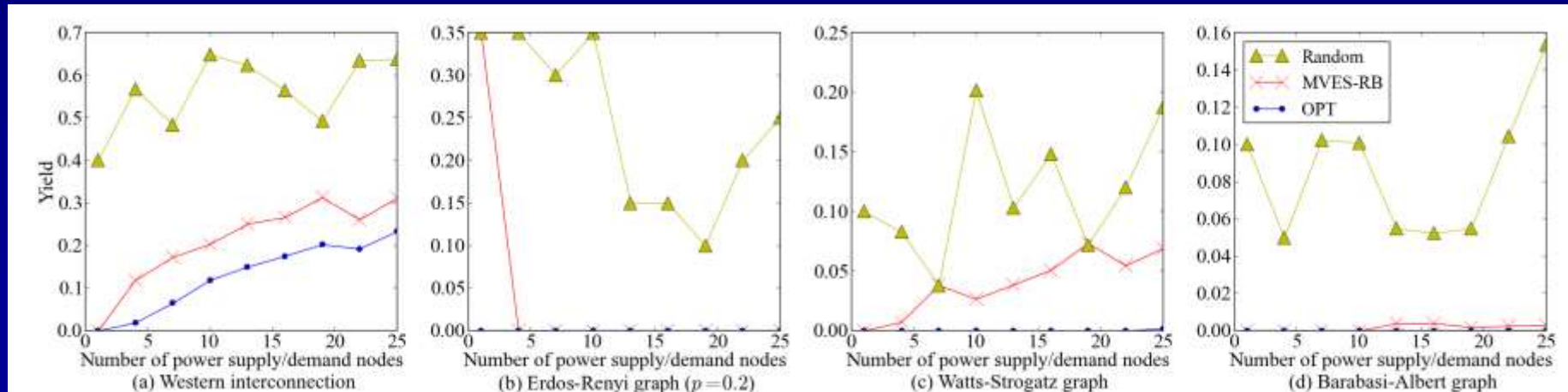
# Vulnerability Analysis - Yield, N-1 Resilient (INFOCOM'14)



The color of each point represents the yield value of a cascade whose epicenter is at that point

# Heuristic Algorithm for Min Yield Problem

- ◆ **Yield:** The ratio between the demand supplied at stabilization and its original value
- ◆ The minimum Yield problem is NP-hard
- ◆ Based on our results, after failure on an edge  $\{p, q\}$ , the flow changes can be bounded by  $|\Delta f_{ij}| \leq \frac{r(p,q)}{1-r(p,q)} |f_{pq}|$
- ◆ Edges with large  $r(p,q)|f_{pq}|$  have large impact on flow changes
- ◆ The algorithm removes edges with large  $r(p,q)|f_{pq}|$



- ◆ The yield at stabilization

# Conclusions

---

- ◆ Cascade propagation models differ from the classical epidemic/percolation-based models
- ◆ Studied properties of the admittance matrix of the grid
- ◆ Derived analytical techniques for studying the impact of a single edge failure
  - Illustrated via simulations and numerical experiments
- ◆ Developed an efficient algorithm to identify the evolution of the cascade
- ◆ Developed a simple heuristic to detect the most vulnerable edges
- ◆ Using the resistance distance and the pseudo-inverse of admittance matrix provides important insights and can support the development of efficient algorithm