Cascading Failures in Power Grids - Analysis and Algorithms

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Cascading Failures in Power Grids

- Power grids rely on physical infrastructure susceptible to physical attacks/failures.
- Failures may cascade.

- An attack/failure will have a significant effect on many interdependent systems (communications, transportation, gas, water, etc.).
IEEE is experiencing significant power disruptions to our U.S. facilities in New Jersey and New York. As a result, you may experience disruptions in service from IEEE.
Physical Attacks/Disasters

- EMP (Electromagnetic Pulse) attack
- Solar Flares - in 1989 the Hydro-Quebec system collapsed within 92 seconds leaving 6 Million customers without power
- Other natural disasters
- Physical attacks


FERC, DOE, and DHS, Detailed Technical Report on EMP and Severe Solar Flare Threats to the U.S. Power Grid, 2010
Sniper Attack on a San Jose Substation, Apr. 2014

Shots in the Dark
A look at the April 16 attack on PG&E’s Metcalf Transmission Substation

1. 12:58 a.m., 1:07 a.m. Attackers cut telephone cables
2. 1:31 a.m. Attackers open fire on substation
3. 1:41 a.m. First 911 call from power plant operator
4. 1:45 a.m. Transformers all over the substation start crashing
5. 1:50 a.m. Attack ends and gunmen leave
6. 1:51 a.m. Police arrive but can’t enter the locked substation
7. 3:15 a.m. Utility electrician arrives

Source: Wall Street Journal
Cascading Failures - Related Work


Cascading failures in the power grid
- Dobson et al. (2001-2010), Hines et al. (2007-2010), Chassin and Posse (2005), Gao et al. (2011),...
- The $N$-$k$ problem where the objective is to find the $k$ links whose failures will cause the maximum damage: Bienstock et al. (2005, 2009)
- Interdiction problems: Bier et al. (2007), Salmeron et al. (2009), ...
- Cascade control: Pfitzner et al. (2011), ...
- Mostly do not consider computational aspects
Outline

- Background
- Power flows and cascading failures
  - Real events, models, and simulations
- Impact of single line failures
- Pseudo-inverse of the admittance matrix and resistance distance
- Efficient algorithm for cascade evolution
- Vulnerability analysis
Power Grid Vulnerability and Cascading Failures

- Power flow follows the laws of physics
- Control is difficult
  - It is difficult to “store packets” or “drop packets”
- Modeling is difficult
  - Final report of the 2003 blackout - cause #1 was “inadequate system understanding” (stated at least 20 times)
- Power grids are subject to cascading failures:
  - Initial failure event
  - Transmission lines fail due to overloads
  - Resulting in subsequent failures
Blackout description (source: California Public Utility Commission)
Event Timeline


15:27:58 to 15:30:00 – CCM tripped in CFE area (needed emergency assistance of 158 MW). IID experienced problems with Imperial Valley-El Centro line resulting in 100MW swing.

15:32:00 to 15:33:44 – IID transformer bank and two units trip. Also two 161 kV lines trip at Niland-WAPA and Niland-Coachella Valley.

15:35:40 to 15:36:45 – Two APS 161 kV lines to Yuma tripped and electrically separated from IID and WAPA. SDG&E now fed power into Yuma area.

15:37:56 – IID's 161 kV tie to WAPA tripped. Import power into Yuma, Imperial Valley, Baja Norte, and San Diego wholly dependant on Path 44.

15:37:58 to 15:38:07 – El Centro Substation (IID) trip due to under frequency. Two units at La Rosita plant (CFE) trip resulting in a loss of 420 MW.

15:38:21 – Path 44 exceeded safety setting of 8000 Amps. Overload relay protection initiated to separate Path 44 between SCE and SDG&E at SONGS switchyard.

15:38:22 to 15:38:38 – SONGS and local power plants trip. 230kV lines open.

15:38:38 – Blackout
Real Cascade - San Diego Blackout

- Failures “skip” over a few hops
- Does not agree with the epidemic/percolation models
Blackout in India, July 2012

- The first 11 line outages leading to the India blackout on July 2012 (numbers show the order of outages)
Power Flow Equations - DC Approximation

- Exact solution to the AC model is infeasible
  \[ f_{ij} = U_i^2 g_{ij} - U_i U_j g_{ij} \cos \theta_{ij} - U_i U_j b_{ij} \sin \theta_{ij} \]
  \[ Q_{ij} = -U_i^2 b_{ij} + U_i U_j b_{ij} \cos \theta_{ij} - U_i U_j g_{ij} \sin \theta_{ij} \]
  and \( \theta_{ij} = \theta_i - \theta_j \).

- We use DC approximation which is based on:
  - \( U_i \equiv 1, \forall i \)
  - \( x_{ij} \) - Load
  - \( \sin \theta_{ij} \approx \theta_{ij} \)
  - \( U_i = 1 \) p.u. for all \( i \)
  - Pure reactive transmission lines - each line is characterized only by its reactance \( x_{ij} = -1/b_{ij} \)
  - Phase angle differences are “small”, implying that \( \sin \theta_{ij} \approx \theta_{ij} \)

- Known as a reasonably good approximation
- Frequently used for contingency analysis
  - Do the assumptions hold during a cascade?
A power flow is a solution \((f, \theta)\) of:

\[
\sum_{v \in N(u)} f_{uv} = p_u, \forall \ u \in V \\
\theta_u - \theta_v - x_{uv} f_{uv} = 0, \forall \ {u, v} \in E
\]

Matrix form:

\[
A \Theta = P
\]

\(A\) is the admittance matrix of the grid defined as:

\[
a_{uv} = \begin{cases} 
0, & u \neq v \text{ and } \{u, v\} \notin E \\
-\frac{1}{x_{uv}}, & u \neq v \text{ and } \{u, v\} \notin E \\
-\sum_{w \in N(u)} a_{vw}, & u = v
\end{cases}
\]

If \(A^+\) is its pseudo-inverse

\[
\Theta = A^+ P
\]

- Load \((p_u < 0)\)
- Generator \((p_u > 0)\)
Different factors can be considered in modeling outage rules

- The main is thermal capacity $u_{ij}$

Simplistic approach: fail lines with $|f_{ij}| > u_{ij}$

*Not part of the power flow problem constraints*

More realistic policy:
Compute the moving average

$$\tilde{f}_{ij} := \alpha |f_{ij}| + (1 - \alpha) \tilde{f}_{ij}$$

$(0 \leq \alpha \leq 1$ is a parameter$)$

**Deterministic outage rule:**
Fail lines with $\tilde{f}_{ij} > u_{ij}$

**Stochastic outage rules**
Cascading Failure Model (Dobson et al.)

\[ P_1 = f_1 = 2000 \text{ MW} \]
\[ P_2 = f_2 = 1000 \text{ MW} \]
\[ P_{13} = 1400 \text{ MW} \]
\[ P_3 = P_3 = 0 \text{ MW} \]

\[ u_{13} = 1800 \text{ MW} \]
\[ x_{13} = 10 \Omega \]

**Until no more lines fail do:**

- Adjust the total demand to the total supply within each component of \( G \)
- Use the power flow model to compute the flows in \( G \)
- Update the state of lines \( \xi_{ij} \) according to the new flows
- Remove the lines from \( G \) according to a given outage rule \( O \)

Initial failure causes disconnection of load 3 from the generators in the rest of the network.

As a result, line (2,3) becomes overloaded.
Numerical Results (Bernstein et al., IEEE INFOCOM'14)

- Obtained from the GIS (Platts Geographic Information System)
- Substantial processing of the raw data
- Used a modified Western Interconnect system, to avoid exposing the vulnerability of the real grid

- 13,992 nodes (substations), 18,681 lines, and 1,920 power stations.
- 1,117 generators (red), 5,591 loads (green)
- Assumed that demand is proportional to the population size
Cascade Development - San Diego area

$N$-Resilient, Factor of Safety $K = 1.2$
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$N$-Resilient, Factor of Safety $K = 1.2 \rightarrow \text{Yield} = 0.33$

For $(N-1)$-Resilient $\rightarrow \text{Yield} = 0.35$

For $K = 2 \rightarrow \text{Yield} = 0.7$

(Yield - the fraction of the demand which is satisfied at the end of the cascade)
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Metrics for Evaluating the Impact of a Single Failure

- Flow Change after failure in the edge $e$
  \[ \Delta f_e = f'_e - f_e \]

- Edge Flow Change Ratio
  \[ S_{e,e'} = \left| \frac{\Delta f_e}{f_e} \right| \]

- Mutual Edge Flow Change Ratio
  \[ M_{e,e'} = \left| \frac{\Delta f_e}{f_{e'}} \right| \]
Graph Used in Simulations

- **Western interconnection**: 1708-edge connected subgraph of the U.S. Western interconnection
- **Erdos-Renyi graph**: A random graph where each edge appears with probability $p = 0.01$
- **Watts and Strogatz graph**: A small-world random graph where each node connects to $k = 4$ other nodes and the probability of rewiring is $p = 0.1$
- **Barabasi and Albert graph**: A scale-free random graph where each new node connects to $k = 3$ other nodes at each step following the preferential attachment mechanism
Effects of a Single Edge Failure

- **Edge Flow Change Ratio**, $S_{e,e'} = \left| \frac{\Delta f_e}{f_e} \right|

- Very large changes in the flow
- Sometimes far from the failure
- There are edges with positive flow increase from zero, far from the initial edge failure
Objective: Compute the mutual edge flow change ratios

Recall that $\Theta = A^+ P$

Method: Update the pseudo-inverse of the admittance matrix upon failure

**Admittance Matrix**: $A$

$$A' = (A + a_{ij} XX^t)$$

**Theorem**: $A'^+ = (A + a_{ij} XX^t)^+ = A^+ - \frac{1}{a_{ij}^{-1} + X^t A^+ X} A^+ X X^t A^+$

*A similar theorem independently proved by Ranjan et al., 2014*
Resistance Distance

- Resistance Distance between nodes $i$ and $j$
  \[ r(i, j) = a_{ii}^+ + a_{jj}^+ - 2a_{ij}^+ \]
- The resistance distance between two nodes is a measure of their connectivity.

All graphs have 1374 nodes
All the edges have reactances equal to 1
Effects of a Single Edge Failure

- Mutual Edge Flow Change Ratio

\[ M_{e,e'} = \left| \frac{\Delta f_e}{f_e} \right| \]

- Using pseudo-inverse of the admittance matrix, \( e = \{i, j\}, e' = \{p, q\} \)

\[ M_{e,e'} = \frac{1 - r(i, p) + r(i, q) + r(j, p) - r(j, q)}{2} \frac{1}{1 - r(p, q)} \]

- Mutual Edge flow change ratios \( (M_{e,e'}) \) are independent of the supply and demand

- Initial Failure
  Represented by a black wide line
Effects of a Single Edge Failure ($M_{e,er}$)

- All graphs have 1374 nodes and each point represents the average of 40 different initial single edge failure events.
Efficient Cascading Failure Evolution Computation

Case I: Failure of an edge which is not a cut-edge

Update $A^+$ after removing the edge $\{1,5\}$, in $O(|V|^2)$
Efficient Cascading Failure Evolution Computation

\[ A' = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix} \]

\[ A'^+ = \begin{bmatrix} 1.2 & 0.4 & -0.2 & -0.6 & -0.8 \\ 0.4 & 0.6 & 0 & -0.4 & -0.6 \\ -0.2 & 0 & 0.4 & 0 & -0.2 \\ -0.6 & -0.4 & 0 & 0.6 & 0.4 \\ -0.8 & -0.6 & -0.2 & 0.4 & 0.2 \end{bmatrix} \]

- **Case II:** Failure of a cut-edge

\[ A'' = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix} \]

- Detect the cut-edge and connected components in \( O(|V|) \)
- Adjust the total demand to equal the total supply within each connected component
- No need to update \( A^+ \)

\[ a'_{23} - 2a'^+_{23} + a'^+_{22} + a'^+_{33} = 0 \]

\[ A'^+_2 - A'^+_3 = \begin{bmatrix} 0.6 & 0.6 & -0.4 & -0.4 & -0.4 \end{bmatrix} \]
The Pseudo-inverse Based Algorithm identifies the evolution of the cascade in $O(|V|^3 + |F_t^*||V|^2)$

Compared to the classical algorithm which runs in $O(t|V|^3)$

If $t = |F_t^*|$ (one edge fails at each round), our algorithm performs $O(\min\{|V|, t\})$ faster
The color of each point represents the yield value of a cascade whose epicenter is at that point.
Heuristic Algorithm for Min Yield Problem

- **Yield**: The ratio between the demand supplied at stabilization and its original value
- The minimum Yield problem is NP-hard
- Based on our results, after failure on an edge \( \{p, q\} \), the flow changes can be bounded by \(|\Delta f_{ij}| \leq \frac{r(p,q)}{1-r(p,q)} |f_{pq}|\)
- Edges with large \( r(p, q)|f_{pq}| \) have large impact on flow changes
- The algorithm removes edges with large \( r(p, q)|f_{pq}| \)

- The yield at stabilization
Conclusions

- Cascade propagation models differ from the classical epidemic/percolation-based models
- Studied properties of the admittance matrix of the grid
- Derived analytical techniques for studying the impact of a single edge failure
  - Illustrated via simulations and numerical experiments
- Developed an efficient algorithm to identify the evolution of the cascade
- Developed a simple heuristic to detect the most vulnerable edges

- Using the resistance distance and the pseudo-inverse of admittance matrix provides important insights and can support the development of efficient algorithm