THEORETICAL APPROACH TO POWER GRID ISLANDING

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Summary

Power Grid Islanding is an effective method to mitigate cascading failures in power grids. The challenge is to partition the power grid network into smaller connected components, called *islands*, so that each island can operate independently for a short period of time. In order for an island to operate, it is necessary that the power supply and demand at that island be almost equal (if the supply and demand are not exactly equal but still relatively close, load shedding/generation curtailing can be used in order for the island to operate). Equality of supply and demand in an island, however, may not be sufficient for its independent operation. It is also important that the infrastructure in that island have the physical capacity to safely carry the power flows. To address this problem, we introduce and study the Doubly Balanced Connected graph Partitioning (DBCP) problem. The DBCP problem is the problem of partitioning a graph into two parts such that both parts are connected and comparable in size, and supply is almost equal to demand in each part. The idea is that when an island is large enough compared to the initial network, it most likely has enough capacity to carry power flows. In this way, the partitions obtained from solutions to the DBCP problem are operational.[1]

Doubly Balanced Connected Graph Partitioning

We introduce and study the Doubly Balanced Connected graph Partitioning (DBCP) problem: Let $G = (V, E)$ be a connected graph with a weight (supply/demand) function $p : V \to \mathbb{Z}$ satisfying $p(V) = \sum_{j \in V} p(j) = 0$. The objective is to partition V into (V_1, V_2) such that $G[V_1]$ and $G[V_2]$ are connected, $|p(V_1)|, |p(V_2)| \leq c_p$, and $\max\{\frac{|V_1|}{|V_2|}, \frac{|V_2|}{|V_1|}\} \leq c_s$, for some constants c_p and c_s. We focus on the case that weights (supply/demand values) are ± 1, but our techniques can be extended, with similar results, to the case in which the weights are arbitrary (not necessarily ± 1), and also to the case that $p(V) \neq 0$ and the excess supply/demand should be split evenly.

The connected partitioning problem with only the size objective has been studied previously. In the most well-known result, Lováz and Gyori [1,3] independently proved that every k-connected graph can be partitioned into k arbitrarily sized connected subgraphs. However, neither of the proofs is constructive, and there are no known polynomial-time algorithms to find such a partition for $k > 3$. The objective of balancing the supply/demand alone, when all $p(i)$ are ± 1, can also be seen as an extension for the objective of balancing the size (which corresponds to $p(i) = 1$).

Since the power grids are designed to withstand a single failure ("$N-1$" standard), and therefore 2-connected, our focus is mainly on the graphs that are at least 2-connected. We use the embedding for k-connected graphs introduced in [2] and show that when G is 2-connected, a solution with $c_p = 1$ and $c_s = 3$ to the DBCP problem always exists and can be found in polynomial time. Moreover, when G is 3-connected, we show that there is always a 'perfect' solution (a partition with $p(V_1) = p(V_2) = 0$ and $|V_1| = |V_2|$, if $|V| \equiv 0 (\text{mod } 4)$), and it can be found in polynomial time.

[1]This abstract summarizes the results that appeared in [4]. This work was supported in part by DTRA grant HDTRA1-13-1-0021, DARPA RADICS under contract #FA-8750-16-C-0054, funding from the U.S. DOE OE as part of the DOE Grid Modernization Initiative, and NSF under grant CCF-1320654 and CCF-1423100.

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