

# OPTIMAL CONTROL OF CASCADING POWER GRID FAILURES WITH IMPERFECT FLOW OBSERVATIONS

Daniel Bienstock, Guy Grebla, Gil Zussman

Columbia University, New York, NY 10027  
{dano@, guy@ee., gil@ee.}columbia.edu

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## Summary

We study control algorithms that stop power grid cascading failures by minimally shedding load (i.e., reducing demand). The control is computed at the beginning of the cascade and applied as the cascade unfolds on the basis of real-time measurements; as the primary focus of this paper we consider an environment where measurements are noisy, missing, or erroneous.

## Introduction

In a cascading failure of a power transmission system, an initial event that disables a possibly small subset of the grid conspires with the laws of physics to set off a sequence of additional outages that, in the worst case, accelerates until a large subset of the network is inoperative, resulting in a significant loss of served power.

The mechanics of the process can be summarized as follows: each time a component of the system fails, a new set of power flows takes hold in the remaining network, following the laws of physics and automatic control actions. Should the new flows, for example, exceed the rating of a given line, then that line will likely fail in the near future. In an adverse scenario this gives rise to a vicious cycle which constitutes the cascade (see e.g., [4, 7]). To protect against a cascade, [6] discusses the design of a robust power transmission system. Control strategies for stopping an ongoing cascade are discussed in [7, 8, 11]. The work in this abstract builds on the model in [5] by incorporating stochasticity; in particular, we model real-time measurement errors.

We consider algorithms that shed load (demand) and curtail supply (generation) as a function of observations taken in real time, with the goal of arresting the cascade with a minimum of demand lost. As a novel contribution, we explicitly model “noise” that would naturally arise in the collection of real-time data. That task relies on a system termed SCADA (“Supervisory control and data

acquisition”) which is physically different from the power transmission system. Under normal operation some of this data is estimated using one of several possible state estimators (see e.g., [9, 12]); in the event of a dangerous cascade, it is quite likely that the measurements conveyed by this system would become susceptible to errors, delays, or loss due to rapidly changing conditions, transients, and possibly even failure of the measurement equipment.

Building on work in [5] we focus on control algorithms that are computed soon after the onset of the cascade, and we assume an initially *slow-moving* cascade so that at the start of the process there is sufficient time (e.g., minutes) to compute an appropriate control algorithm; once computed, the control will be applied as the cascade unfolds.

In devising a load-shedding schedule to respond to a potential cascade, one must decide when, where, and by how much demand is to be shed. Our method can be viewed as a data-driven approach for computing such actions – it is data-driven because it relies on the knowledge of the initial event, and on the real-time measurements performed to apply the control. Moreover our algorithm seeks to handle measurement error – we explicitly assume that measurements can be incorrect, and yet we look for a control that minimizes lost demand subject to (effectively) a norm constraint on the errors. To this effect, in this abstract we rely on a method akin to the Sample Average Approximation Method [10] to generate, a priori, an appropriate sample of measurement error sequences, and to optimize a control over that set of sequences.

## Models for Power Flows, Cascades, and Controls

We consider the linearized approximation model for the power grid [2, 3]. In the linearized approximation (or DC approximation) model we are given a directed graph  $G$  with  $n$  nodes and  $m$  arcs (corresponding, respectively, to buses and lines). The physics of power flow transmission

is described by two systems of equations,

$$Nf = \beta, \quad N^T \phi - Xf = 0, \quad (1)$$

Here  $N$  denotes the node-arc incidence matrix of  $G$  [1],  $\beta$  is an  $n$ -vector indicating (net) supply at each bus,  $X = \text{diag}\{x_{ij}\}$ , and where for each line  $(i, j)$ ,  $x_{ij} > 0$  is the reactance. The vector  $f$  indicates power flows; the vector  $\phi$  indicates *phase angles*. See, e.g., [2, 3].

We consider a cascade and control actions over a duration of  $T$  discrete time-steps. As the cascade progresses and control actions are taken, the (functioning) power grid graph changes due to line outages. In this work, we focus on a deterministic outage rule

$$\text{line } j \text{ becomes outaged if } f_j > u_j. \quad (2)$$

We denote  $G^t$  the power grid at the beginning of time-step  $t$ . The cascade and control operations are described in Framework 1, where we assume that in step 4 the grid measurements are obtained with a non-deterministic error bounded by some  $b \geq 0$ .

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### Framework 1 Cascade Control

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**Input:** a power grid with graph  $G$ . Set  $G^1 = G$ .

- 1: **Compute** control algorithm
  - 2: **For**  $t = 1, 2, \dots, T - 1$   $\triangleright$  controlled time-step  $t$  of the cascade
  - 3:   Set  $f^t =$  vector of power flows in  $G^t$
  - 4:   Obtain grid measurements
  - 5:   **Apply control**
  - 6:   Set  $g^t =$  vector of resulting power flows in  $G^t$
  - 7:   Set  $\mathcal{O}^t =$  set of lines of  $G^t$  that become outaged in time-step  $t$
  - 8:   Set  $G^{t+1} = G^t - \mathcal{O}^t$
  - 9:   Adjust loads and generation in  $G^t$
  - 10: **Termination** (time-step  $T$ ). If any island of  $G^T$  has line overloads, proportionally shed demand in that island until all line overloads are eliminated.
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### Control Methods

We use an affine-law-based control. We will compute triples  $s^t, b^t, c^t$ , for  $t = 1, \dots, T$ , and at time step  $t$ , we will scale all demands in each *component* (or “island”)  $K$  of  $G^t$  by the common multiplier  $0 \leq \lambda_K^t \leq 1$  defined by the expression

$$\lambda_K^t \doteq \min\{1, [b^t + s^t(c^t - \hat{\kappa}_K^t)]^+\}, \quad (3)$$

where  $\hat{\kappa}_K^t$  is the *observed* maximum line overload of any line in component  $K$ . We will write  $\hat{\kappa}_K^t = \kappa_K^t + \epsilon_K^t$ , where  $\kappa_K^t$  is the *actual* (or exact) and  $\epsilon_K^t$  is the *error*. We assume

$\epsilon_K^t$  is a random variable with distribution  $P$  and that  $\epsilon_{K1}^{t1}$  and  $\epsilon_{K2}^{t2}$  are i.i.d. for every  $t1, t2, K1, K2$ .

Assume without loss of generality that initially the grid is connected (there is one component) and let  $\epsilon$  be the vector of all errors over the  $T$  rounds. Denote by  $Y^A(\epsilon, \beta)$  the yield that a control algorithm  $A$  obtains for a power grid with initial demand vector  $\beta$  and measurements errors  $\epsilon$ , at the end of the termination step. Note that  $Y^A(\epsilon, \beta)$  is a random variable. Our goal is to develop a control algorithm  $A$  that either maximizes the *expected yield*  $\mathbb{E}_P(Y^A(\epsilon, \beta))$  or provides certain lower bound guarantee for it. Such control algorithm will provide an efficient way to cope with a cascade and its nondeterministic affects on the accuracy of collected grid measurements.

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