ANALYSIS OF FAILURES IN THE POWER GRID VIA THE PSEUDO-INVERSE OF THE ADMITTANCE MATRIX

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SIAM Workshop on Network Science 2014
July 6-7 · Chicago

Summary

Recent large-scale power outages demonstrated the limitations of percolation- and epidemic-based tools in modeling failures in power grids. Hence, we study the impact of line failures on the flow changes and cascade in the transmission system of the power grid by using a linearized power flow model. We use the pseudo-inverse of the power grid admittance matrix to obtain upper bounds on the flow changes after a failure, develop an efficient algorithm to identify the cascading failure evolution, and develop a simple heuristic to find a set of line failures with the highest impact. Overall, the results demonstrate that the resistance distance and the pseudo-inverse of admittance matrix provide important insights and can support the development of efficient algorithms.

Model

We adopt the linearized (or DC) power flow model, which is widely used as an approximation for the AC power flow model [1]. We represent the power grid by an undirected graph $G = (V, E)$ where V and E are corresponding to the buses and transmission lines, respectively. p_v is the active power *supply* ($p_v > 0$) or *demand* ($p_v < 0$) at node $v \in V$ (for a *neutral node* $p_v = 0$). We assume *pure reactive* lines, where each edge $\{u, v\}$ is characterized by its *reactance* $x_{uv} = x_{vu}$. A *power flow* is a solution (f, θ) of:

$$\sum_{v \in N(u)} f_{uv} = p_u,\ \forall\ u \in V \tag{1}$$

$$\theta_u - \theta_v - x_{uv} f_{uv} = 0,\ \forall\ \{u, v\} \in E \tag{2}$$

where $N(u)$ is the set of neighbors of node u, f_{uv} is the power flow from node u to node v, and θ_u is the phase angle of node u. Eq.(1)-(2) are equivalent to the matrix equation: $A\Theta = P$, where $\Theta \in \mathbb{R}^{|V|\times 1}$ is the vector of phase angles, $P \in \mathbb{R}^{|V|\times 1}$ is the power supply/demand vector, and $A = [a_{ij}] \in \mathbb{R}^{|V|\times|V|}$ is the *admittance matrix* of the graph G. The key to the results below is that power flow equations can be solved by using the *Moore-Penrose Pseudo-inverse* of the admittance matrix, $A^+ = [a^+_{ij}]$ [4].

To study the effects of a *single edge (e′) failure after one round*, we define the ratio between the change of flow on an edge, e, and the initial flow on the failed edge, e', as *mutual edge flow change ratio*: $M_{e,e'} = |\Delta f_e / f_{e'}|$.

We also use the cascading failure model of [2].

Failure Impact and Cascade Evolution

The following theorem provides an analytical rank-1 update of the pseudo-inverse of the admittance matrix.

Theorem 1. *If $\{i, j\}$ is not a cut-edge, then,*

$$A'^+ = (A + a_{ij} X X^t)^+ = A^+ - \frac{1}{a_{ij}^{-1} + X^t A^+ X} A^+ X X^t A^+$$

in which X is an $n \times 1$ vector with 1 in i^{th} entry, -1 in j^{th} entry, and 0 elsewhere.

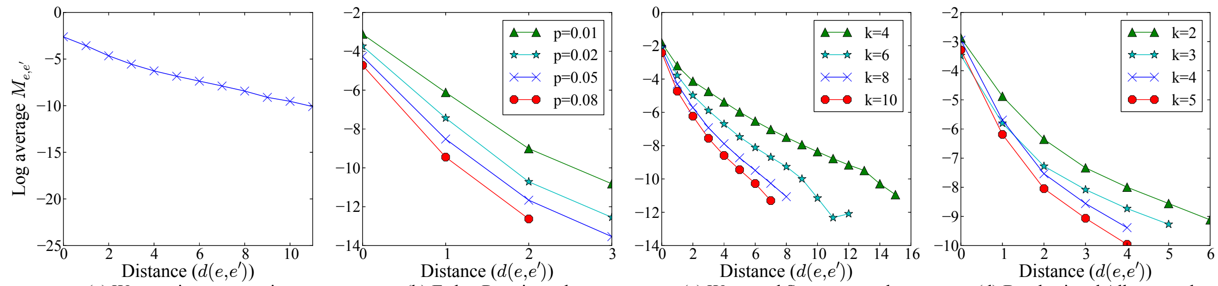
For simplicity, we assume that $x_{uv} = 1\ \forall\{u, v\} \in E$. In this case, the admittance matrix A is the *Laplacian matrix* of the graph. We use Theorem 1 and the notion of *resistance distance* to provide a formula for computing the flow changes and mutual edge flow change ratios after a single edge failure.

Definition. *The* resistance distance between two nodes $i, j \in V$ *is* $r(i, j) := a^+_{ii} + a^+_{jj} - 2a^+_{ij}$.

Lemma 1. *The flow change and the mutual edge flow change ratio for an edge $e = \{i, j\} \in E$ after a failure in a non-cut-edge $e' = \{u, v\} \in E$ are,*

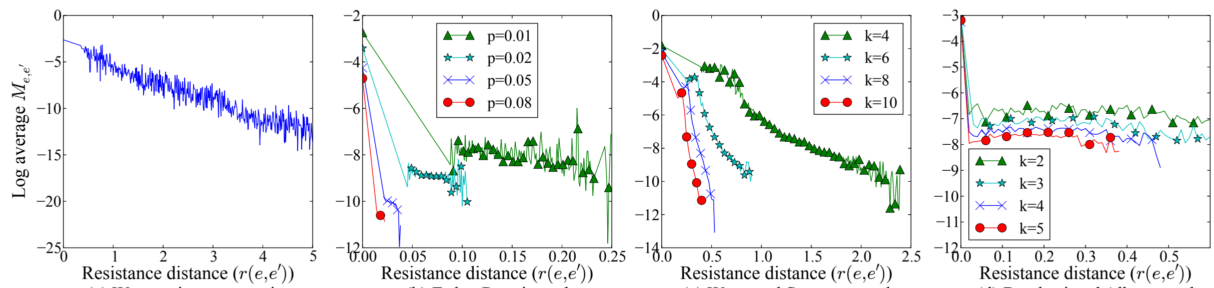
$$\begin{aligned} \Delta f_{ij} &= \frac{1}{2} \frac{-r(i,u) + r(i,v) + r(j,u) - r(j,v)}{1 - r(u,v)} f_{uv}, \\ M_{e,e'} &= \frac{1}{2} \frac{-r(i,u) + r(i,v) + r(j,u) - r(j,v)}{1 - r(u,v)}. \end{aligned}$$

This abstract summarizes some of the results that appear in [4]. This work was supported in part by CIAN NSF ERC under grant EEC-0812072, and DTRA grant HDTRA1-13-1-0021.



(a) Western interconnection (b) Erdos-Renyi graph (c) Watts and Strogatz graph (d) Barabasi and Albert graph

Figure 2: The average of the mutual flow change ratios ($M_{e,e'}$) versus the distance from the initial edge failure for different graph classes and a subgraph of the Western interconnection with 1374 nodes. Each point represents the average of 40 different initial single edge failure.



(a) Western interconnection (b) Erdos-Renyi graph (c) Watts and Strogatz graph (d) Barabasi and Albert graph

Figure 3: The average of the mutual flow change ratios ($M_{e,e'}$) versus the resistance distance from the initial edge failure. Each point represents the average of 40 different initial single edge failure events. For clarity, the markers appear for every 5 data points.

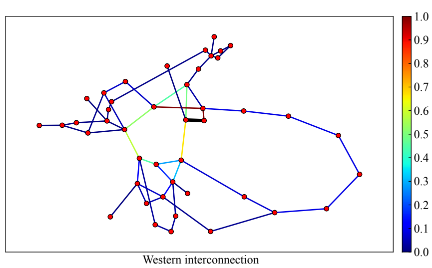


Figure 1: The mutual edge flow change ratios ($M_{e,e'}$) after an edge failure (represented by black wide line) in a 44 node subgraph of the Western interconnection.

The Lemma implies that mutual edge flow change ratios are independent of the power supply/demand distribution and solely depend on the grid structure.

Corollary 1 below provides an upper bound on the flow changes after a failure in a non-cut-edge.

Corollary 1. *The flow changes in any edge* $e = \{i, j\} \in E$ *after a failure in a non-cut-edge* $e' = \{u, v\} \in E$ *are bounded by,*

$$|\Delta f_{ij}| \le \frac{r(u,v)}{1 - r(u,v)} |f_{uv}|, \;\; M_{e,e'} \le \frac{r(u,v)}{1 - r(u,v)}.$$

Using Lemma 1, Fig. 1 illustrates the mutual edge flow change ratios after an edge failure. Figs. 2 and 3 show the mutual edge flow change ratio ($M_{e,e'}$) as the function of distance ($d(e,e')$) and resistance distance ($r(e,e')$) from the failure.

Once lines fail, there is a need for low complexity algorithms to control and mitigate the cascade [2,3]. Hence, using Theorem 1 we develop the low complexity *Cascading Failure Evolution – Pseudo-inverse Based (CFE-PB) Algorithm* for identifying the evolution of a cascade that may be initiated by a failure of *several* edges [4]. The running time is $O(|V|^3 + |F_t^*||V|^2)$ ($|F_t^*|$ and t are the number of edges that eventually fail and cascade rounds), which is $O(\min\{|V|, t\})$ times lower than that of [2].

Finally, from Corollary 1, it seems that edges with large $r(i,j) \times |f_{ij}|$ have greater impact on the flow changes on the other edges. Based on this, we introduce a very simple heuristic, termed the Most Vulnerable Edges Selection – Resistance distance Based (MVES-RB) Algorithm, for finding a set of edge failures with the largest impact at the end of the cascade [4].

References

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