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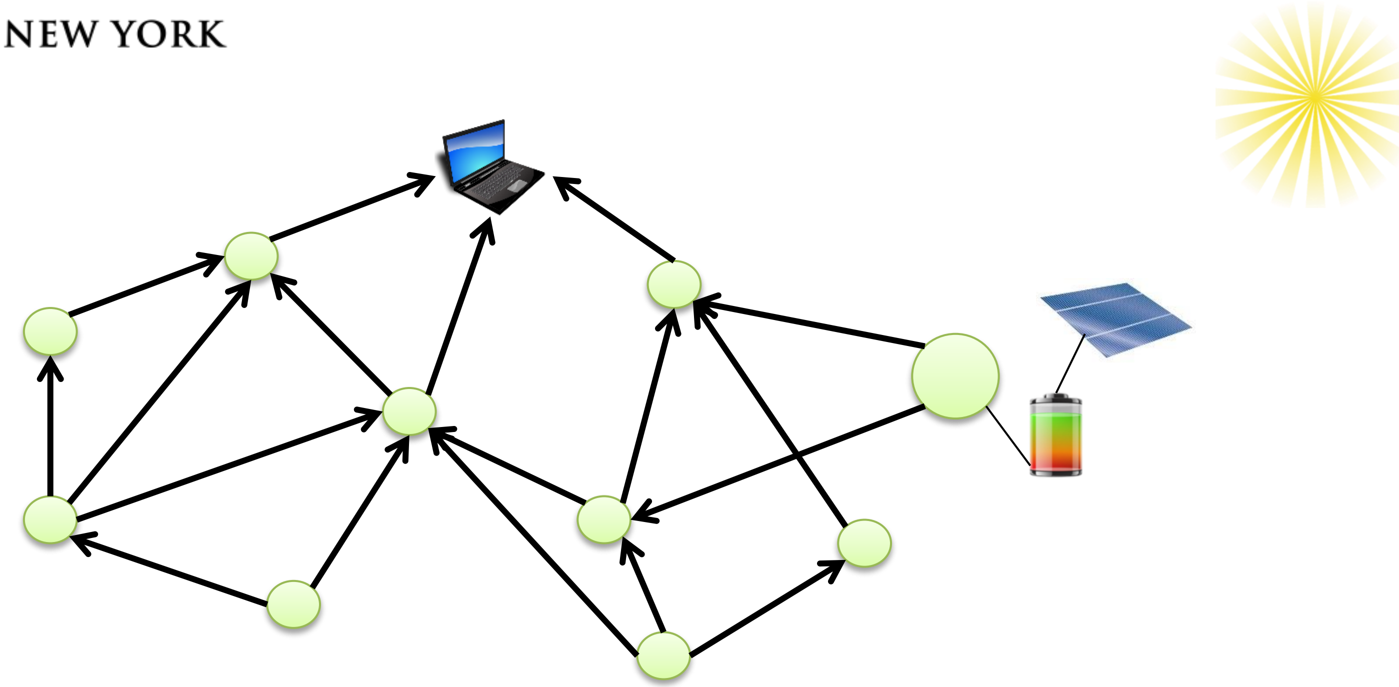

Max-min Fair Rate Allocation and Routing in Energy Harvesting Networks: Algorithmic Analysis

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Internet of Things

- Internet of Things (IoT)/Internet of Everything (IoE):
  - Networking devices and objects that traditionally have not been networked.

industrial internet CONSORTIUM

at&t
CISCO
GE
intel
IBM

ALLSEEN ALLIANCE

Haier
LG
Panasonic
Microsoft
QUALCOMM
Silicon Image
SHARP
technicolor
TP-LINK

Academia

Columbia University in the City of New York
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Berkeley University of California
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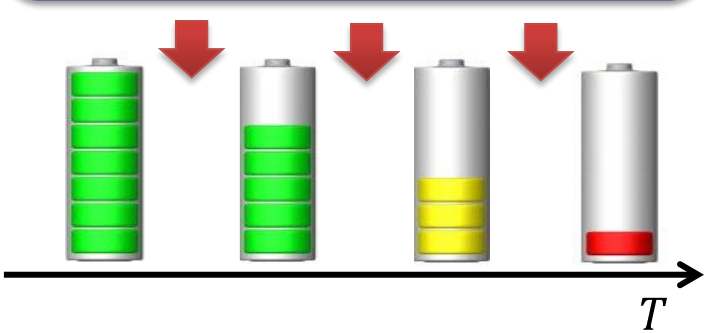
Energy Harvesting Networks

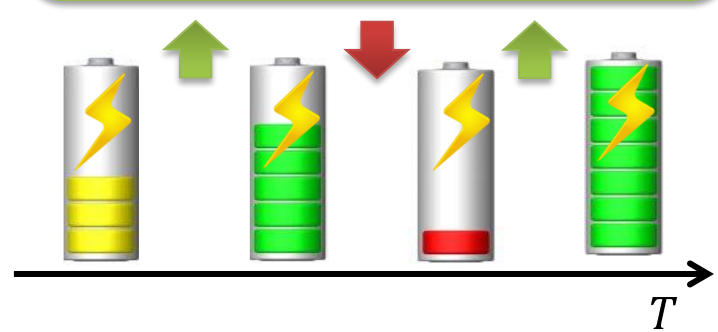
- One of the main enablers for IoT
- Small self-powered devices with rechargeable batteries
- Environmental energy harvesting (solar, wind, kinetic, RF)
- Energy is spent on: sensing, transmitting, and receiving data
- Applications: sensing, monitoring, tracking, etc.

- Paradigm shift in sensor networks:

Classical sensor networks:
Maximize lifetime

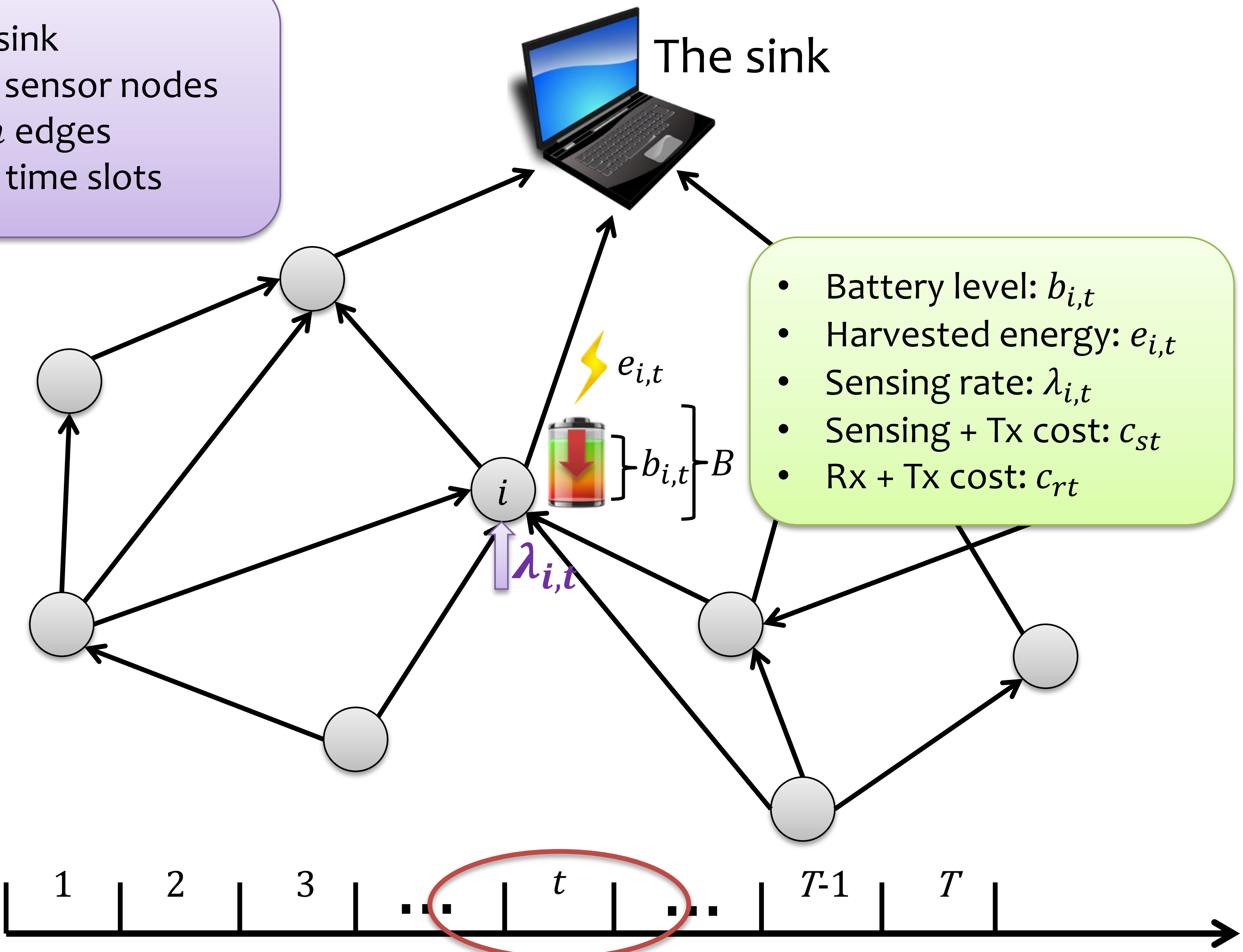


Energy harvesting networks:
Enable perpetual operation



The Model

- 1 sink
- n sensor nodes
- m edges
- T time slots



- Battery level: $b_{i,t}$
- Harvested energy: $e_{i,t}$
- Sensing rate: $\lambda_{i,t}$
- Sensing + Tx cost: c_{st}
- Rx + Tx cost: c_{rt}

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- n sensor nodes
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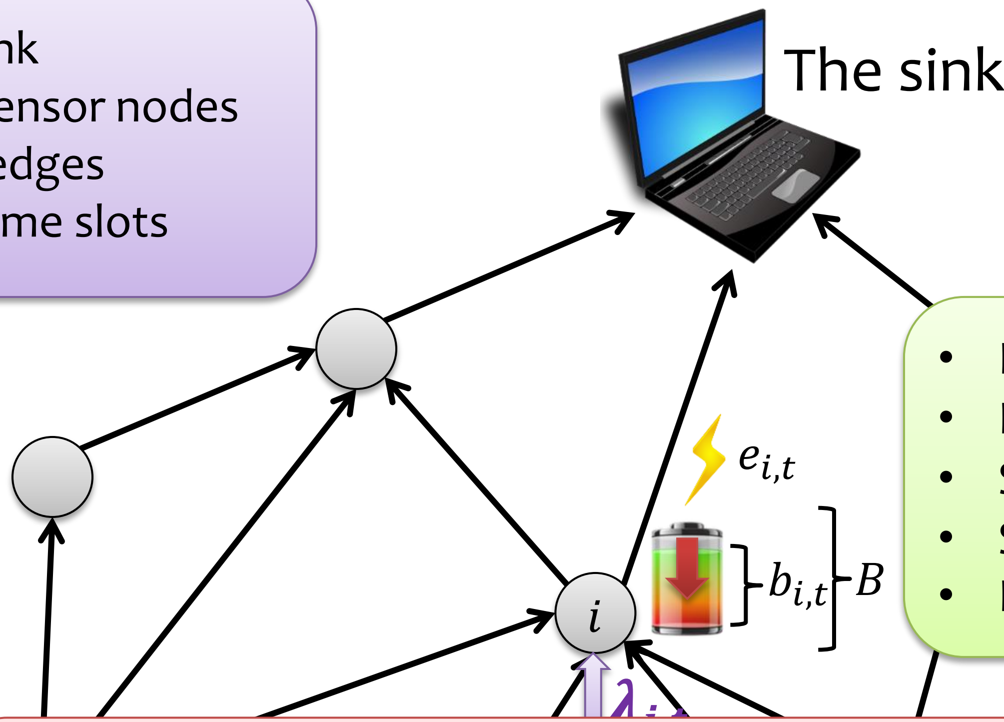
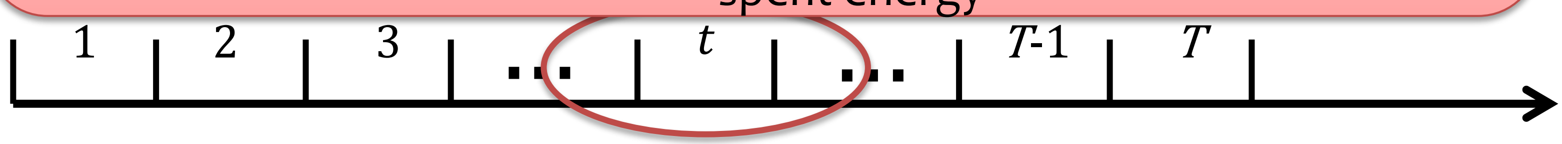


- Battery level: $b_{i,t}$
- Harvested energy: $e_{i,t}$
- Sensing rate: $\lambda_{i,t}$
- Sensing + Tx cost: c_{st}
- Rx + Tx cost: c_{rt}

$$(flow\ in)_{i,t} + \lambda_{i,t} = (flow\ out)_{i,t}$$

$$b_{i,t+1} = \begin{cases} B, & \text{if } \ b_{i,t} + e_{i,t} - \left(c_{st}\lambda_{i,t} + c_{rt}(flow\ in)_{i,t}\right) > B \\ b_{i,t} + e_{i,t} - \underbrace{\left(c_{st}\lambda_{i,t} + c_{rt}(flow\ in)_{i,t}\right)}_{\text{spent energy}}, & \text{otherwise} \end{cases}$$



Overview of the Results

- Rate assignment and routing algorithm design:
  - Centralized;
  - Finite time horizon & predictable energy profile;
  - Fairness—required over both the nodes and the time;

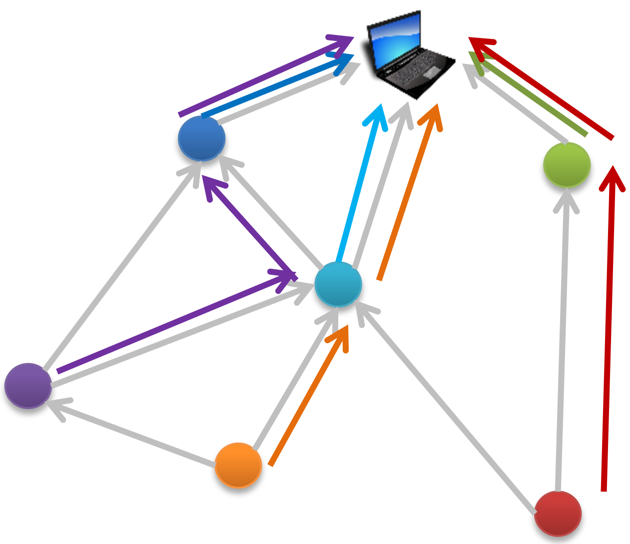
Water-filling framework implementation

	Maximizing	Fixing	Total	Rates	Routing
Single-path	$\tilde{O}(nT)$	$O(mT)$	$\tilde{O}(nmT^2)$	✓	
Fixed fractional	$\tilde{O}(\max(T, MF(n,m)))$	$O(m)$	$\tilde{O}(n(T + MF(n,m)))$	✓	✓
Time-variable fractional	$\tilde{O}(T^2/\varepsilon^2 \cdot (nT + MCF(n,m)))$	$LP(nT, mT)$	$\tilde{O}(nT(T^2/\varepsilon^2 (nT + MCF(n,m)) + LP(nT, mT))$	✓	✓

Overview of the results

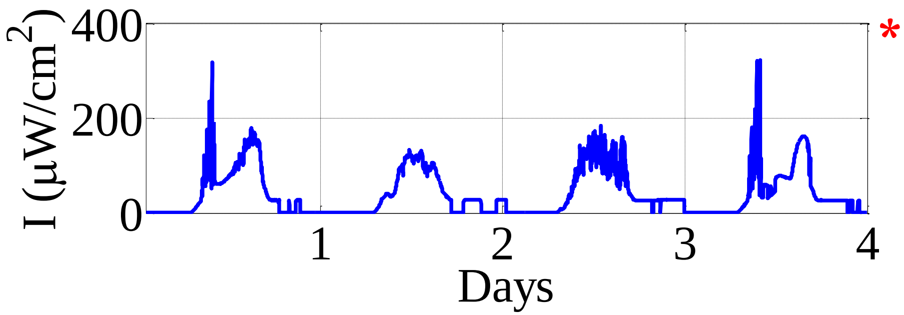
- Computing single-path routes is "**hard**" even for $T = 1$:
  - Tree: NP-hard to approximate within $\log(n)$
  - Single path: NP-hard to compute

- However, for fixed single-path routing:
  - Designed an algorithm that determines the routes that maximize the minimum rate



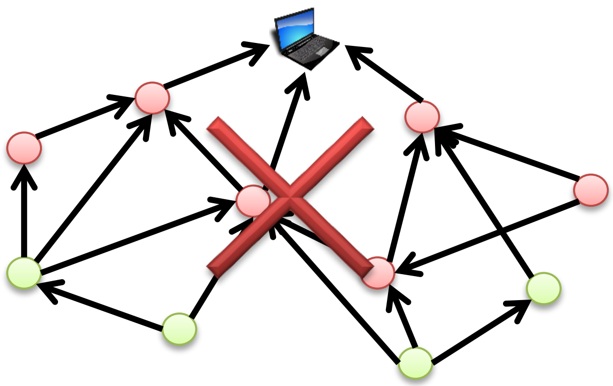
Network Operation and Fairness

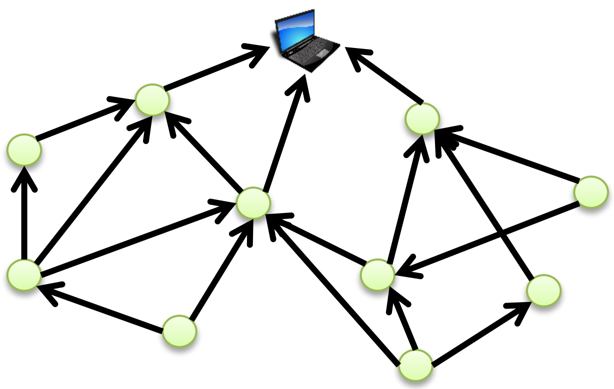
- Perpetual operation: fairness over time



*enhants.ee.columbia.edu

- Balanced data acquisition: fairness over nodes





The Problems

Definition. *An assignment* $\{\lambda_{i,t}\}, i \in [n], t \in [T]$*, is said to be max-min fair if no* $\lambda_{i,t}$ *from the set can be increased without either losing feasibility, or lowering another* $\lambda_{j,\tau} \leq \lambda_{i,t}$*.*

Assuming: known initial battery levels $b_{i,1}$ and harvested energies $e_{i,t}$

Requiring max-min fairness of the rates $\{\lambda_{i,t}\}$, determine:

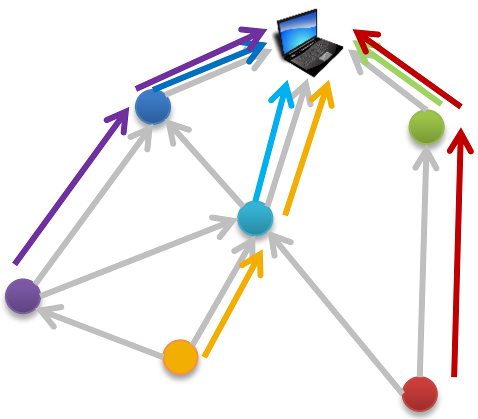
- **the routing** of the required type, and
- **the rate assignment** $\{\lambda_{i,t}\}$

Note:

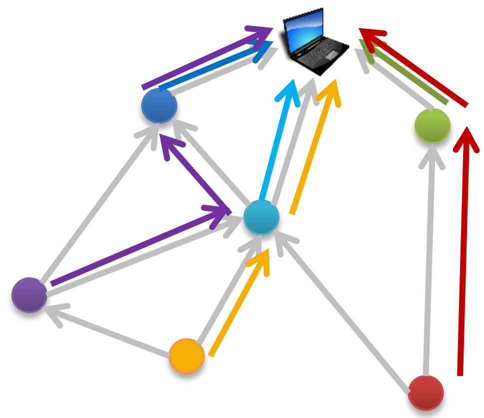
- *Fairness is required over both the nodes and the time*
- *The goal is to understand algorithmic properties of the problem*

Routing Types

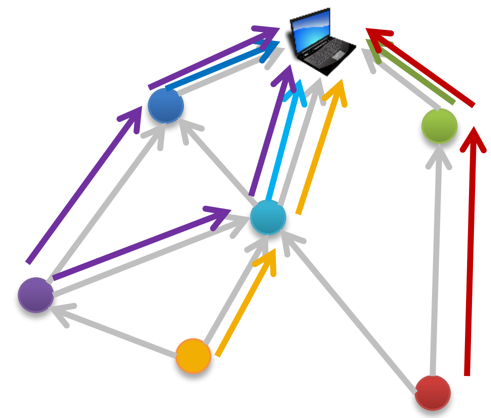
Routing Tree



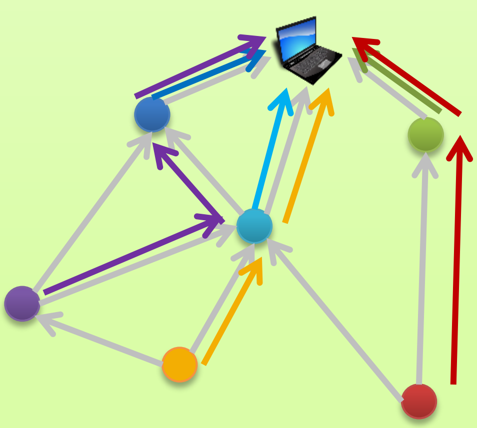
Single Path Routing



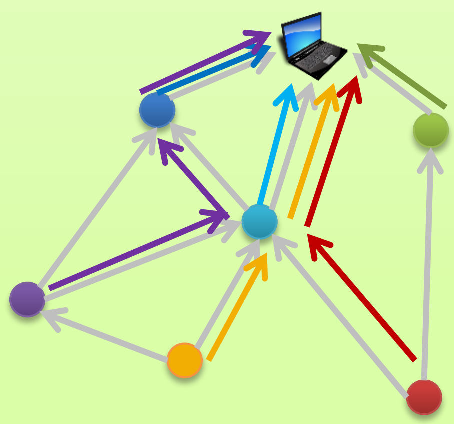
Multi-Path Routing



Time-variable routing:

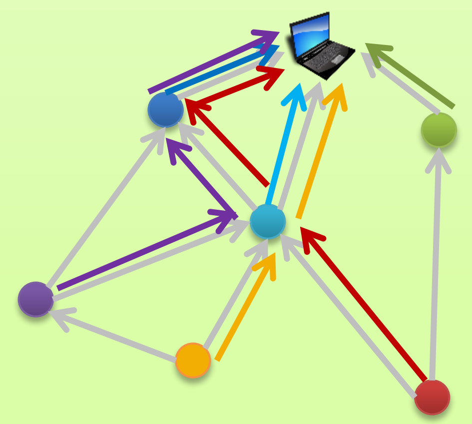


$t = 1$



$t = 2$

…



$t = T$

Related Work

Energy harvesting networks:

- Node or a link; e.g., [Gorlatova et al. 2013], [Srivastava & Koksal 2013], [Ozel et al. 2011]
- Network—control-theoretic approach: optimize time-averages—*time unfair*! E.g., [Gatzianas et al. 2010], [Huang & Neely 2013], [Mao et al. 2012]
- Most relevant to our work:
  - [Gurakan et al. 2013] (two hops)
  - [Liu et al. 2011] (constant rates)

Sensor networks:

- Maximum lifetime routing: [Chang & Tassiulas 2004], [Madan & Lall 2006], …

Equivalent to maximizing lowest rate for $T = 1$.

- Determining a max lifetime tree: [Buragohain et al. 2005]

Implies the NP-hardness of finding a tree. We show hardness of approximation.

Unsplittable (single path) max-min fair routing:

- Bottleneck routing: [Bertsekas & Gallager 1992], [Charny et al. 1995], …

Much simpler: unit costs, static capacities.

- Unsplittable routing: [Kleinberg et al. 1999]

Implies our hardness results for single path routing.

Fractional max-min fair routing:

- Traditional network flows: [Megiddo 1974]

Much simpler: unit energy costs, static capacities.

- LP framework: [Radunović, Le Boudec 2007]

Requires a huge number of large LPs.

- Sensor networks: [Chen et al. 2007]

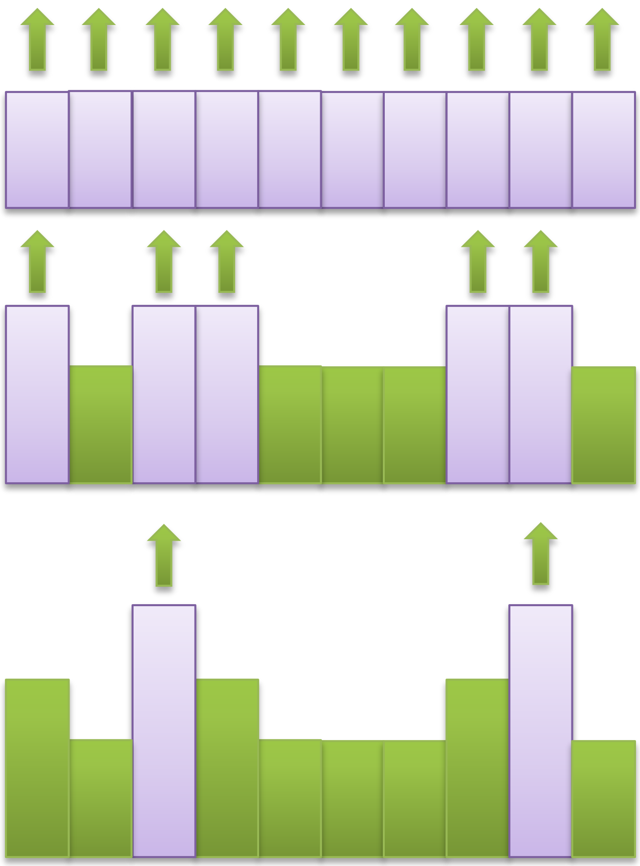
Simpler problem—static capacities.

- Energy harvesting networks: [Liu et al. 2011]

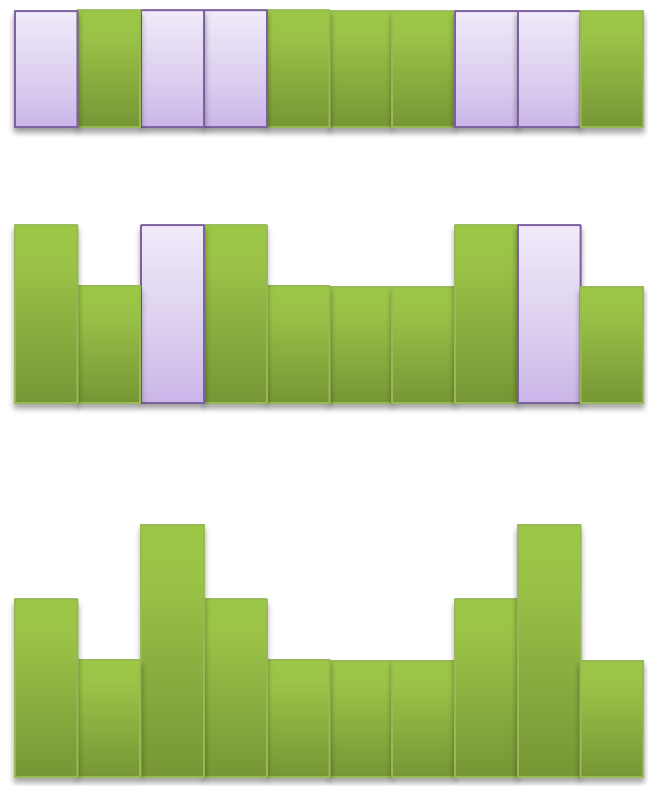
Simpler problem: constant rates; heuristic.

Water-filling Framework and Rate Assignment

Maximization?

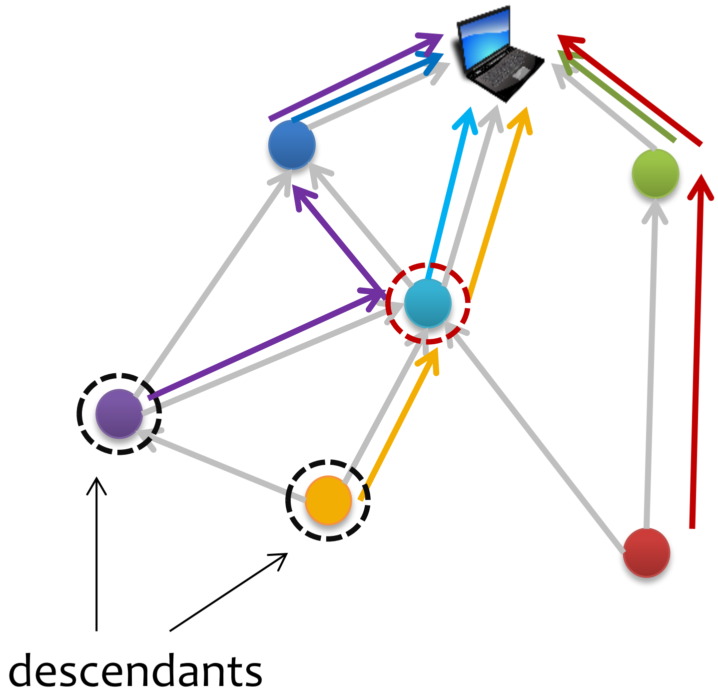


Fixing?



Single-Path Routing: Rate Assignment

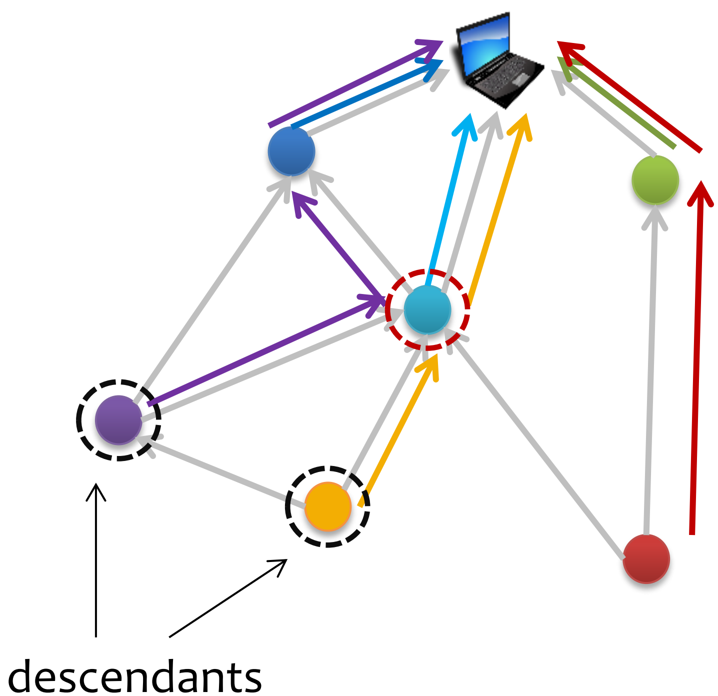
- Assuming that the routing is given at the input, determine the max-min fair rate assignment



Maximization:

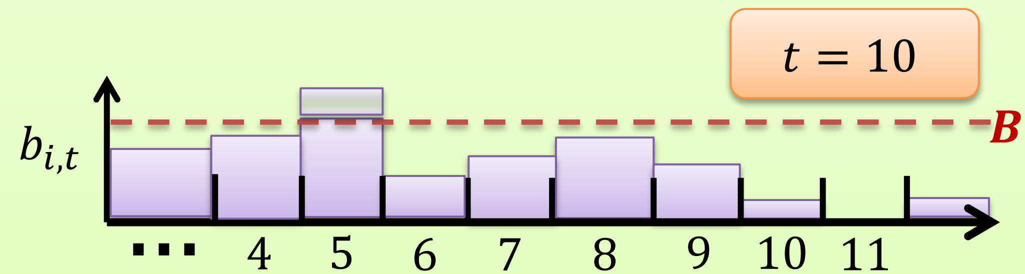
1. For each node i, find the maximum supported rate, assuming i's descendants can support the same rate
2. Return the minimum rate from 1.

Single-Path Routing: Rate Assignment



Fixing:

1. Fix all $\lambda_{i,t}$'s for which $b_{i,t+1} = 0$
2. Fix the rates $\lambda_{i,\tau}$ in all the slots with no extra energy preceding $b_{i,t+1} = 0$

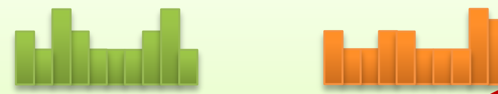


3. Fix the rates off all the descendants of i's fixed in 1. and 2., in the same time slots

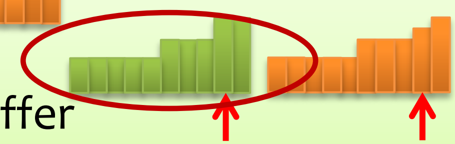
Single-Path Routing: Determining Routes

- A **"good"** routing:
  - The routing that provides lexicographically maximum rate assignment

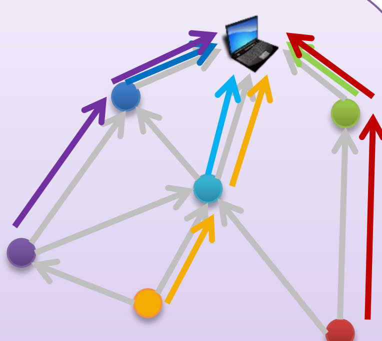
Lexicographic comparison of two vectors:



1. Order the elements of both vectors in non-decreasing order
2. Going from left to right, find the first element in which they differ
3. The vector with the higher element is lexicographically higher

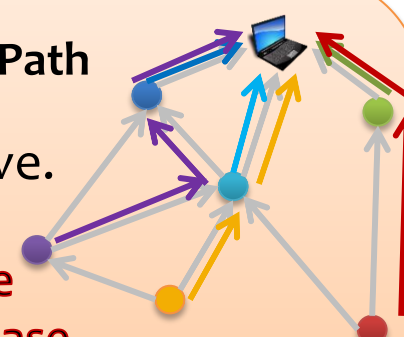


Results: Tree



Hard to approximate within log(n) even for a single time slot.

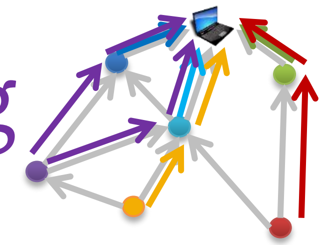
Results: Single Path



NP-hard to solve.

However, for the time-invariable case, designed a combinatorial algorithm for a relaxed case that only maximizes the min rate.

Time-variable Fractional Routing



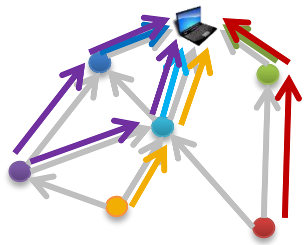
- Restructuring constraints get a packing problem

$$\sum_{\tau=1}^{t} c_{st}\lambda_{i,\tau} + c_{rt}(f\ in)_{i,\tau} \leq b_{i,1} + \sum_{\tau=1}^{t} e_{i,\tau}, \qquad \text{for } t \in \{1, 2, \dots, T\}$$

$$\sum_{\tau=s}^{t} c_{st}\lambda_{i,\tau} + c_{rt}(f\ in)_{i,\tau} \leq B + \sum_{\tau=s}^{t} e_{i,\tau}, \qquad \text{for s} \in \{2, \dots, T\}, t \in \{s+1, \dots, T\}$$

- Feasible rates: at least as hard as feasible 2-commodity flow
  - Unlikely to be solved optimally without linear programming
- PTAS design:
  - Maximization: packing algorithm [Plotkin et al. 1995] + structural properties
  - Fixing: 1 LP over ε-neighborhood of the solution after maximization

Fixed Fractional Routing



- Observation:
  - Each node spends a fixed amount of energy per slot

Pre-processing:

- Determine the max Δb_i that a node can spend per slot
- $\lambda_i = 0$

Maximization:

- For $\lambda \in [0, \min_i \Delta b_i]/c_{st}$, via binary search:
  - If λ_i not fixed:
    - Set supply flow to $\lambda_i = \lambda_i + \lambda$
    - Set capacity of node i to $(\Delta b_i - c_{st}\lambda)/c_{rt}$
  - Solve feasible flow
- $\Delta b_i = \Delta b_i - c_{st}\lambda_i$

Fixing:

- Fix λ_i if and only if i has no directed path to the sink in the residual graph

Summary & Future Work

- Algorithmic study of max-min fairness in energy harvesting networks
- Benchmarking or centralized solution for highly-predictable energy profiles
- Generalized flow problems—might be of independent interest
- Insights into the problem structure

- Fairness guarantees with a:
  - Distributed algorithm?
  - Online algorithm?

  \+ low communication overhead
- Different types of fairness:
  - Proportional fairness?
  - α-fairness?

Water-filling framework implementation

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Single-path	$\tilde{O}(nT)$	$O(mT)$	$\tilde{O}(nmT^2)$	✓	
Fixed fractional	$\tilde{O}(\max(T, MF(n,m)))$	$O(m)$	$\tilde{O}(n\max(T, MF(n,m)))$	✓	✓
Time-variable fractional	$\tilde{O}(T^2/\varepsilon^2 \cdot (nT + MCF(n,m)))$	$LP(nT, mT)$	$\tilde{O}(nT(T^2/\varepsilon^2 (nT + MCF(n,m)) + LP(nT, mT))$	✓	✓

Thanks!

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