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Quantifying the Effect of k-line Failures in Power Grids

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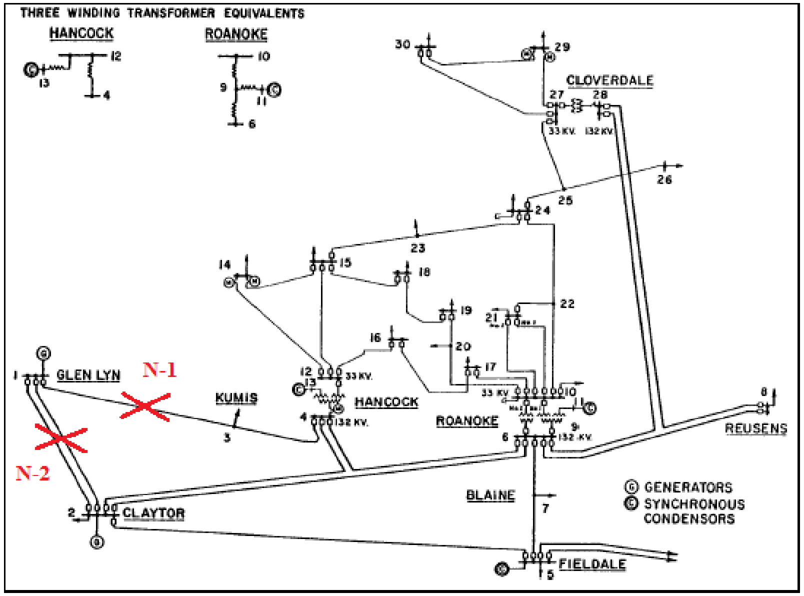
Contingency Analysis

- $N-1$ (insufficient)
- $N-2$ (recently)
- Provide a method to detect *critical cases* for $N-k$ contingency analysis *efficiently*



* Figure is from "Studies of Contingencies in Power Systems through a Geometric Parameterization Technique, Part II: Performance Evaluation" by Bonini Neto et. al.

- We define the *disturbance value* of a failure to quantify the effect of k-line failures
- Show that it can be computed in $O(1)$ time

DC Power Flow

- Given the supply/demand vector $\vec{p} \in \mathbb{R}^{n\times 1}$ a *power flow* is a solution $\vec{f}$ and $\vec{\theta}$ of:

$$A\vec{\theta} = \vec{p}$$
$$YD^t\vec{\theta} = \vec{f}$$

Solution

$$\vec{\theta} = A^+\vec{p}$$
$$YD^tA^+\vec{p} = \vec{f}$$

*$D \in \{-1,0,1\}^{n\times m}$: the *incidence matrix of the graph G* representing the grid

**$Y \in \mathbb{R}^{m\times m}$: the diagonal matrix of admittance values

***$A = DYD^t$: the admittance matrix, and A^+ is its pseudo-inverse

- Define $R := D^tA^+D$ as the *matrix of equivalent reactance values*

Failure Analysis

- *Lemma:* If G' represents the grid after a k-line failure in lines 1 to k, then

$$A'^{+} = A^{+} + A^{+} D_k Y_{k|k}^{1/2} \left[I - Y_{k|k}^{1/2} D_k^t A^{+} D_k Y_{k|k}^{1/2} \right]^{-1} Y_{k|k}^{1/2} D_k^t A^{+}$$

*Q_k: the submatrix of Q limited to the first k columns

**$Q_{k|k}$: the submatrix of Q limited to the first k columns and rows

***$\bar{k}$: the indices other than 1 to k

- Define *k-line outage distribution matrix $\mathcal{L}$:*

$$\mathcal{L} := Y_{\bar{k}|\bar{k}} R_{\bar{k}|k} Y_{k|k}^{1/2} \left[I - Y_{k|k}^{1/2} D_k^t A^{+} D_k Y_{k|k}^{1/2} \right]^{-1} Y_{k|k}^{-1/2}$$

$$\Delta \vec{f}_{\bar{k}} = \mathcal{L} \vec{f}_k$$

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Disturbance value

- Define $\sum_{i=k+1}^{m} y_{ii}^{-1} \Delta f_i^2$ as the *disturbance value*
- $\Delta \vec{f}_{\bar{k}}^{\,t} \, Y_{\bar{k}|\bar{k}}^{-1} \Delta \vec{f}_{\bar{k}} = \sum_{i=k+1}^{m} y_{ii}^{-1} \Delta f_i^2$ reflects both
  1. big phase difference changes
  2. big flow changes
- By using extensive linear algebra techniques,

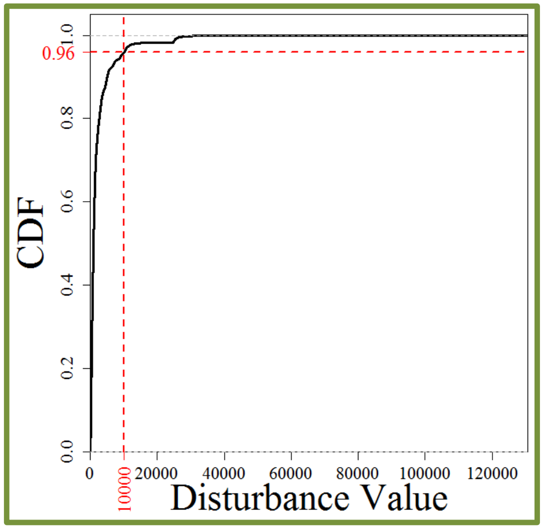
$$\Delta \vec{f}_{\bar{k}}^{\,t} \, Y_{\bar{k}|\bar{k}}^{-1} \Delta \vec{f}_{\bar{k}} = \underbrace{\vec{f}_k^{\,t} Y_{k|k}^{-1/2} \left[-I + \left(I - Y_{k|k}^{\frac{1}{2}} R_{k|k} Y_{k|k}^{\frac{1}{2}} \right)^{-1} \right] Y_{k|k}^{-1/2} \vec{f}_k}_{\text{Can be computed in } O(k^3) \approx O(1)}$$

Can be computed in $O(k^3) \approx O(1)$

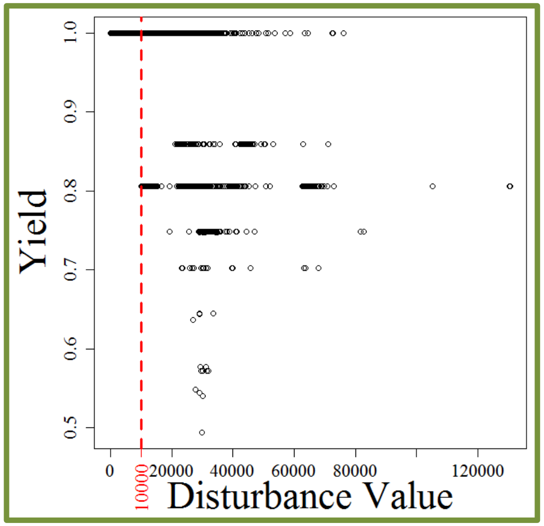
Numerical Results/Conclusions

- Disturbance values provide a clear separation between the important/less important cases
- Less that 4% of the total cases (3-line failures) are critical for $N - 3$ contingency analysis

CDF of the disturbance values for all 3-line failures in IEEE 118-bus system



Yield* versus disturbance value for all the cascades initiated by 3-line failures in IEEE 118-bus system



*Yield: the ratio between the demand supplied at the end of a cascade and the original demand after an initial failure event