SNR Analysis for Multi-Rate UWB-IR

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***Abstract*— This letter is motivated by the need to design MAC protocols for impulse radio ultrawide bandwidth systems. It evaluates the signal-to-noise ratio (SNR) in such *multi-rate* systems in which the different users transmit with different rates and different power levels. The SNR is obtained for all combinations of pulse-position modulation or pulse-amplitude modulation, with time-hopping and/or direct-sequence spreading. We show that the SNR in a multi-rate time-hopping system is different from a single-rate system, and is highly affected by the low-rate users.**

***Index Terms*— Ultrawide bandwidth (UWB), Medium Access Control (MAC), signal-to-noise ratio (SNR), impulse radio, multi-rate systems.**

I. Introduction

ULTRAWIDE bandwidth (UWB) is an emerging technology that has the potential to become the new enabler for low-cost low-power wireless networks. Specifically, UWB impulse radio (IR) [1] is a physical layer technique that uses a time-hopping signal composed of subnanoseconds pulses (referred to as monocycles). Due to the distinct characteristics of UWB-IR, reusing medium access control (MAC) protocols originally designed for narrow-band systems may be inefficient [2]. Currently, only a few aspects of the UWB MAC layer have been been studied (e.g. [3], [4]). For example, [5]–[8] formulated optimization problems that attempt to capture the particularities of the physical layer and use these as a basis for a MAC protocol design.

These optimization approaches build upon the results of [1], regarding multi-user communications in UWB-IR. Interestingly, although the proposed MAC protocols are supposed to allocate different data rates to the different users, [1] considered only a *single-rate* system, where all the users transmit at the same data rate. Hence, in order to enable an optimization that will lead to a rate allocation protocol, there is a need to obtain results regarding *multi-rate* systems. In this letter, we extend the work of [1] to include multi-rate transmissions over frequency-flat channels. In particular, we consider multiple transmitters with different power levels and different data rates. We study the combinations of pulse-position modulation (PPM) or pulse-amplitude modulation (PAM), with time-hopping (TH) and/or direct-sequence (DS) signaling. We obtain lower bounds on the SNR by treating monocycle collisions at the physical layer in a similar manner to the treatment of MAC layer collisions.

Manuscript received August 7, 2006. The associate editor coordinating the review of this letter and approving it for publication was Gianluca Mazzini. This work was supported in part by a Belgian American Educational Foundation fellowship, a Marie Curie Fellowship within the 6th EC Framework Programme, ONR Young Investigator Award N00014-03-1-0489 and Grant N000140610064, NSF Grants ANI-0335256 and ECS-0636519, and the Draper Laboratory Robust Distributed Sensor Networks Program.

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Digital Object Identifier 10.1109/LCOMM.2007.061233.

II. System Description

We consider a system similar to the UWB-IR system analyzed in [1]. In an UWB-IR system, the transmission of a single data bit corresponds to sending N_{s} monocycles over the physical channel and recombining these at the receiver (N_{s} is the processing gain). Each monocycle is of duration T_{w} and is transmitted within a chip whose duration is T_{c}. A frame is composed of N_{h} chips and its length is denoted by T_{f} ($N_{\mathrm{h}}T_{\mathrm{c}} \leq T_{\mathrm{f}}$). For each frame, the TH sequence determines the location of the chip in which the transmission takes place. We denote the number of users (active links) by N_{u}. When all users have the same N_{s} and T_{f}, the data rate of a user is $R = 1/(N_{\mathrm{s}}T_{\mathrm{f}})$.

In this letter we consider a multi-rate system. Theoretically, the rate on a link can be changed by varying N_{s} or T_{f} [9]. We will only consider the more practical case in which different links have different values of N_{s}. We denote the processing gain of user k by $N_{\mathrm{s}}^{(k)}$ and the data rate of user k by $R^{(k)} = 1/(N_{\mathrm{s}}^{(k)}T_{\mathrm{f}})$. We focus on user 0 and assume that the processing gains of the different users satisfy $N_{\mathrm{s}}^{(k)}/N_{\mathrm{s}}^{(0)} \in \mathbb{N}$ or $N_{\mathrm{s}}^{(0)}/N_{\mathrm{s}}^{(k)} \in \mathbb{N}$. The unit-energy transmit block $p^{(k)}(t)$ for user k, consisting of $N_{\mathrm{s}}^{(k)}$ monocycles is given by

$$p^{(k)}(t) = \frac{1}{\sqrt{N_{\mathrm{s}}^{(k)}}} \sum_{n=0}^{N_{\mathrm{s}}^{(k)}-1} d_n^{(k)} w\left(t - c_n^{(k)}T_{\mathrm{c}} - nT_{\mathrm{f}}\right)$$

where $w(t)$ is the monocycle, $d_n^{(k)} \in \{-1, 1\}$ represents a DS, and $c_n^{(k)} \in \{0, 1, \ldots, N_{\mathrm{h}} - 1\}$ represents a TH sequence. For a pure TH system, $d_n^{(k)} = 1, \forall n, k$. Considering both PPM and PAM, the transmitted waveform from user k can be written as

$$s^{(k)}(t) = \sqrt{\frac{P_{\mathrm{avg}}^{(k)}}{R^{(k)}}} \sum_{l=-\infty}^{+\infty} a_l^{(k)} p^{(k)}\left(t - lT^{(k)} - b_l^{(k)}\Delta\right)$$

where $P_{\mathrm{avg}}^{(k)}$ is the average transmit power, $a_l^{(k)} \in \{-1, 1\}$ and $b_l^{(k)} \in \{0, 1\}$ are the PAM and PPM mappings of the l-th data bit of the k-th user, $T^{(k)} = 1/R^{(k)}$ is the bit duration, and Δ is a fixed multiple of T_{w} ($\Delta < T_{\mathrm{c}}$).

We assume that the system is designed such that no intersymbol interference (ISI) occurs. The signal of the k-th transmitter $s^{(k)}(t)$ propagates through the physical channel $h^{(k)}(t) = \alpha^{(k)}\delta\left(t - \tau^{(k)}\right)$, where $\alpha^{(k)}$ models the attenuation associated with the propagation path and $\tau^{(k)}$ represents the asynchronism between the clock of the signal received from transmitter k and the receiver clock. Ignoring antenna effects, the signal at the input of the receiver is given by $r(t) = \sum_{k=0}^{N_{\mathrm{u}}-1} \alpha^{(k)} s^{(k)}\left(t - \tau^{(k)}\right) + v(t)$, where $v(t)$ is a wideband AWGN process with power spectral density $N_0/2$, encapsulating thermal noise and non-UWB interference. We introduce $q^{(k)}(t) = \alpha^{(k)} p^{(k)}\left(t - \tau^{(k)}\right)$ and perform single user detection as follows. The signal $r(t)$ is filtered by $q^{(0)}(-t)$

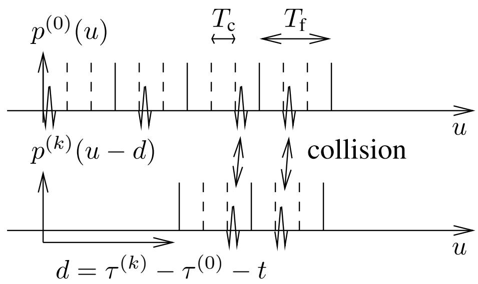

Fig. 1. An example of transmissions by two users: user 0 and k with $N_s^{(0)}=4$ and $N_s^{(k)}=2$. The 3rd (resp. 4th) monocycle of user 0 collides with the 1st (resp. 2nd) monocycle of user k. Hence $\mathbf{C}^{(0,k)}(t)$ is a 4×2 matrix with a one on row 3, column 1 and row 4, column 2.

for PAM transmission or by $q^{(0)}(-t)-q^{(0)}(-t-\Delta)$ for PPM transmission. Then, the filtered signal is sampled at time $mT^{(0)}$ to obtain a decision statistic for the m-th bit of user 0 (i.e., for $a_m^{(0)}$ or $b_m^{(0)}$): $z_m=\sum_{k=0}^{N_u-1}\sqrt{\frac{P_{avg}^{(k)}}{R^{(k)}}}\phi_m^{(k)}+n_m$, where $\sqrt{\frac{P_{avg}^{(k)}}{R^{(k)}}}\phi_m^{(k)}$ is the contribution of user k to the decision statistic (an explicit expression for $\phi_m^{(k)}$ will be given in Section III). For PAM, $\mathbb{E}\{|n_m|^2\}=\frac{N_0}{2}\left|\alpha^{(0)}\right|^2$, while for PPM, $\mathbb{E}\{|n_m|^2\}=N_0\left|\alpha^{(0)}\right|^2$ [9]. Our goal is to derive the SNR:

$$\mathrm{SNR}_0=\frac{\frac{P_{avg}^{(0)}}{R^{(0)}}\mathbb{E}\left\{\left|\phi_m^{(0)}\right|^2\right\}}{\mathbb{E}\left\{|n_m|^2\right\}+\mathbb{E}\left\{\left|\sum_{k=1}^{N_u-1}\sqrt{\frac{P_{avg}^{(k)}}{R^{(k)}}}\phi_m^{(k)}\right|^2\right\}}. \quad (1)$$

It is easily verified that $\mathbb{E}\{\phi_m^{(k)}\}=0$ and that, due to the independence of the different transmitters, the multiple access interference (MAI) can be written as

$$\mathbb{E}\left\{\left|\sum_{k=1}^{N_u-1}\sqrt{\frac{P_{avg}^{(k)}}{R^{(k)}}}\phi_m^{(k)}\right|^2\right\}=\sum_{k=1}^{N_u-1}\frac{P_{avg}^{(k)}}{R^{(k)}}\mathbb{E}\left\{\left|\phi_m^{(k)}\right|^2\right\}.$$

III. Statistical Characterization of MAI

A. Preliminaries

We define $\mathbf{C}^{(0,k)}(t)$ as a *collision matrix* between the monocycles composing a bit transmitted by user 0 and those composing a bit transmitted by user k. For a time t, we delay the k-th user's monocycle over a time $d=\tau^{(k)}-\tau^{(0)}-t$. Fig. 1 illustrates an example of the transmissions of users 0 and k. The elements of $\mathbf{C}^{(0,k)}(t)$ are random variables defined such that $\mathbf{C}_{i,j}^{(0,k)}(t)=1$ when the i-th monocycle of user 0 collides with the j-th monocycle of user k, and zero otherwise. Accordingly, the dimensions of $\mathbf{C}^{(0,k)}(t)$ are $N_s^{(0)}\times N_s^{(k)}$. $\mathbf{C}^{(0,k)}(t)$ depends on the TH sequences of the different users and on their relative delays. Since T_f is constant, $\mathbf{C}^{(0,k)}(t)$ always contains at most a single non-zero entry per column and per row. We define

$$\rho^{(0,k)}(t)=\int_{-\infty}^{+\infty}q^{(0)}(u)\,q^{(k)}(t+u)\,du =\frac{\alpha^{(0)}\alpha^{(k)}\beta^{(0,k)}}{\sqrt{N_s^{(0)}N_s^{(k)}}}\left(\mathbf{d}^{(0)}\right)^T\mathbf{C}^{(0,k)}(t)\,\mathbf{d}^{(k)}$$

where the vectors $\mathbf{d}^{(0)}$, $\mathbf{d}^{(k)}$ are the DS's of users 0 and k (their dimensions are $N_s^{(0)}$ and $N_s^{(k)}$, respectively) and $\beta^{(0,k)}$ is a constant that depends on the difference $\tau^{(k)}-\tau^{(0)}$ indicating the amount of overlap between the colliding monocycles. We will abbreviate $\alpha^{(0)}\alpha^{(k)}\beta^{(0,k)}/\sqrt{N_s^{(0)}N_s^{(k)}}$ by $A^{(0,k)}$.

We now reformulate the PPM decision statistic so that it can be interpreted as a PAM decision statistic. For PAM,

$$\phi_m^{(k)}=\sum_{l=-\infty}^{+\infty}a_l^{(k)}\rho^{(0,k)}\left(mT^{(0)}-lT^{(k)}\right) \quad (2)$$

while for PPM,

$$\phi_m^{(k)}=\sum_{l=-\infty}^{+\infty}\Big\{\rho^{(0,k)}\left(mT^{(0)}-lT^{(k)}-\Delta b_l^{(k)}\right) -\rho^{(0,k)}\left(mT^{(0)}-lT^{(k)}-\Delta\left(b_l^{(k)}-1\right)\right)\Big\}. \quad (3)$$

Note that for a given $b_l^{(k)}$, only one of the two terms in (3) can be non-zero. This allows us to rewrite $\phi_m^{(k)}$ as

$$\phi_m^{(k)}=\sum_{l=-\infty}^{+\infty}\tilde{a}_l^{(k)}\tilde{\rho}^{(0,k)}\left(mT^{(0)}-lT^{(k)}\right)$$

with $\tilde{a}_l^{(k)}\in\{-1,+1\}$. Now, $\tilde{\rho}^{(0,k)}\left(mT^{(0)}-lT^{(k)}\right)$ has *exactly* the same statistical properties as $\rho^{(0,k)}\left(mT^{(0)}-lT^{(k)}\right)$. Hence, we can analyze PPM and PAM in a unified way.

In the following section, we obtain an upper bound on the MAI by assuming that the users are synchronized (i.e., $\tau^{(k)}=\tau^{(0)}$, and therefore $\beta^{(0,k)}=1\,\forall k$). User k is then maximally correlated with user 0, resulting in an interference caused by user k which is greater than the actual interference. When the l-th bit of user k collides with the m-th bit of user 0, the synchronization assumption allows us to characterize the matrix $\mathbf{C}^{(0,k)}\left(mT^{(0)}-lT^{(k)}\right)$. In that case, it can be horizontally or vertically partitioned into square sub-matrices whose dimensions are $N_s^{\min}=\min\{N_s^{(0)},N_s^{(k)}\}$. In all the sub-matrices but one, all the elements are equal to 0. In a single sub-matrix the elements of the diagonal are either 0 or 1 (representing collisions in the different frames). Since the TH codes are i.i.d., we define $C_i\sim\mathrm{Bernoulli}(\gamma)$, $\forall i\in\{1,\ldots,N_s^{\min}\}$ as i.i.d. random variables representing the diagonal elements of this sub-matrix, where γ denotes the probability of a monocycle collision in a frame ($\gamma=1/N_h$). All the non-diagonal elements of this sub-matrix are 0.

B. Time-Hopping

For a system without DS, $\mathbf{d}^{(0)}=\mathbf{1}$ and $\mathbf{d}^{(k)}=\mathbf{1}$, and therefore, $\left(\mathbf{d}^{(0)}\right)^T\mathbf{C}^{(0,k)}\left(mT^{(0)}-lT^{(k)}\right)\mathbf{d}^{(k)}$ is equal to $\sum_{i=1}^{N_s^{\min}}C_i$. Namely, it is a Binomial random variable representing the *total number of collisions* experienced by the m-th bit of user 0 due to the l-th bit of user k. By using the second moment of the Binomial distribution we get

$$\mathbb{E}\left\{\left|\left(\mathbf{d}^{(0)}\right)^T\mathbf{C}^{(0,k)}\left(mT^{(0)}-lT^{(k)}\right)\mathbf{d}^{(k)}\right|^2\right\} =\gamma N_s^{\min}\left\{1+\gamma(N_s^{\min}-1)\right\}\approx\gamma N_s^{\min}\left\{1+\gamma N_s^{\min}\right\}.$$

Due to the dependence on $N_s^{\min}$, we separately consider users with lower rate than user 0 and users with higher rate than user 0.

$$\mathrm{SNR}_0 = \frac{P_{\mathrm{avg}}^{(0)}\left|\alpha^{(0)}\right|^2}{R^{(0)}}\left\{\frac{N_0}{2}+\left\{\sum_{k\in S_{\mathrm{L}}^{(0)}} P_{\mathrm{avg}}^{(k)}\left|\alpha^{(k)}\right|^2\left(\frac{\gamma^2}{R^{(0)}}+T_{\mathrm{f}}\gamma\right)+\sum_{k\in S_{\mathrm{H}}^{(0)}} P_{\mathrm{avg}}^{(k)}\left|\alpha^{(k)}\right|^2\left(\frac{\gamma^2}{R^{(k)}}+T_{\mathrm{f}}\gamma\right)\right\}\right\}^{-1} \quad (8)$$

Low-rate user: When $N_{\mathrm{s}}^{(k)} = L^{(k)} N_{\mathrm{s}}^{(0)}$ ($L^{(k)} \in \mathbb{N}$), only a single bit of user k interferes with the bit of user 0, and therefore, due to (2)

$$\mathbb{E}\left\{\left|\phi_m^{(k)}\right|^2\right\} \approx \left(A^{(0,k)}\right)^2 \gamma N_{\mathrm{s}}^{(0)}\left\{1+\gamma N_{\mathrm{s}}^{(0)}\right\}. \quad (4)$$

High-rate user: When $N_{\mathrm{s}}^{(0)} = L^{(k)} N_{\mathrm{s}}^{(k)}$ ($L^{(k)} \in \mathbb{N}$), exactly $L^{(k)}$ bits of user k will interfere with the bit of user 0, and therefore, for some l':

$$\phi_m^{(k)} = A^{(0,k)} \sum_{l=l'}^{l'+L^{(k)}} a_l^{(k)}\left(\mathbf{d}^{(0)}\right)^T \mathbf{C}^{(0,k)}\left(mT^{(0)}-lT^{(k)}\right)\mathbf{d}^{(k)}$$

leading us to

$$\mathbb{E}\left\{\left|\phi_m^{(k)}\right|^2\right\} \approx \left(A^{(0,k)}\right)^2 \gamma L^{(k)} N_{\mathrm{s}}^{(k)}\left\{1+\gamma N_{\mathrm{s}}^{(k)}\right\}. \quad (5)$$

C. Direct-Sequence/Time-Hopping

In a hybrid DS/TH system with i.i.d. sequences, when the l-th bit of user k collides with the m-th bit of user 0

$$\begin{aligned}\mathbb{E}&\left\{\left|\left(\mathbf{d}^{(0)}\right)^T \mathbf{C}^{(0,k)}\left(mT^{(0)}-lT^{(k)}\right)\mathbf{d}^{(k)}\right|^2\right\}\\ &= \mathbb{E}\left\{\sum_{i=1}^{N_{\mathrm{s}}^{\min}}\sum_{j=1}^{N_{\mathrm{s}}^{\min}} d_i^{(0)} d_j^{(0)} C_i C_j d_i^{(k)} d_j^{(k)}\right\}\\ &= \mathbb{E}\left\{\sum_{i=1}^{N_{\mathrm{s}}^{\min}} \left(C_i\right)^2\right\} = \gamma N_{\mathrm{s}}^{\min}.\end{aligned}$$

Low-rate user: When $N_{\mathrm{s}}^{(k)} = L^{(k)} N_{\mathrm{s}}^{(0)}$ ($L^{(k)} \in \mathbb{N}$),

$$\mathbb{E}\left\{\left|\phi_m^{(k)}\right|^2\right\} \approx \left(A^{(0,k)}\right)^2 \gamma N_{\mathrm{s}}^{(0)}. \quad (6)$$

High-rate user: When $N_{\mathrm{s}}^{(0)} = L^{(k)} N_{\mathrm{s}}^{(k)}$ ($L^{(k)} \in \mathbb{N}$), in a similar manner to the derivation of (5), we find that

$$\mathbb{E}\left\{\left|\phi_m^{(k)}\right|^2\right\} \approx (A^{(0,k)})^2 \gamma L^{(k)} N_{\mathrm{s}}^{(k)} = (A^{(0,k)})^2 \gamma N_{\mathrm{s}}^{(0)}. \quad (7)$$

IV. SNR Evaluation and Discussions

We now obtain a lower bound on the SNR after despreading. We focus on PAM and note that under PPM the results have an additional factor of two in the noise power.

TH: We define $S_{\mathrm{L}}^{(0)} \subset \{1,\ldots,N_{\mathrm{u}}-1\}$ as the set of users with a rate lower than (or equal to) user 0, and $S_{\mathrm{H}}^{(0)}$ as the set of users with a rate higher than user 0. Substituting (4) and (5) into (1), we obtain (8), shown at the top of this page.

DS/TH: Substituting (6) and (7) into (1), we find

$$\mathrm{SNR}_0 = \frac{P_{\mathrm{avg}}^{(0)}\left|\alpha^{(0)}\right|^2}{R^{(0)}}\left\{\frac{N_0}{2}+T_{\mathrm{f}}\gamma\sum_{k=1}^{N_{\mathrm{u}}-1} P_{\mathrm{avg}}^{(k)}\left|\alpha^{(k)}\right|^2\right\}^{-1}. \quad (9)$$

It can be seen that when all users use the same data rate, (8) and (9) have the same form, very similar to eq. (44) from [1]. For a multi-rate system, the system *without* DS leads to a more involved SNR expression. In such a system, interference from a low-rate user does not depend on its rate, since it adds up constructively. For a high-rate user, less interference is created due to the averaging effect of the data bits. The higher the rate, the more averaging occurs, the less that user causes interference. On the other hand, in a DS system, the interference caused by a user does not depend on the data rate.

V. Conclusions

We obtained lower bounds on the SNR in multi-rate UWB-IR systems. The SNR equations differ from previous results and can form the basis of optimization problems that provide insight into how rate and power should be allocated in UWB-IR networks. Extension to a multi-code multi-rate scheme [10] is achieved by treating the different data streams of a user as coming from virtual users. Extensions to include the (possibly user-dependent) code rate of an error-correcting code are straightforward. Extension of this work to a multi-path channel remains a topic for further research.

Acknowledgments

We thank E. Modiano for helpful comments and suggestions.

References

[1] M. Z. Win and R. A. Scholtz, "Ultra-wide bandwidth time-hopping spread-spectrum impulse radio for wireless multiple-access communications," *IEEE Trans. Commun.*, vol. 48, no. 4, pp. 679-691, Apr. 2000.
[2] B. Radunovic and J. Y. Le Boudec, "Optimal power control, scheduling and routing in UWB networks," *IEEE J. Sel. Areas Commun.*, vol. 22, no. 7, pp. 1252-1270, Sept. 2004.
[3] M.-G. Di Benedetto, L. De Nardis, M. Junk, and G. Giancola, "(UWB)^2: uncoordinated, wireless, baseborn, medium access control for UWB communication networks," *ACM/Springer MONET*, vol. 10, no. 5, pp. 663-674, 2005.
[4] R. Merz, J. Widmer, J.-Y. Le Boudec, and B. Radunovic, "A joint PHY/MAC architecture for low-radiated power TH-UWB wireless ad-hoc networks," *Wireless Commun. and Mobile Comput.*, vol. 5, no. 5, pp. 567-580, 2005.
[5] Y. Chu and A. Ganz, "A centralized MAC protocol for QoS support in UWB-based wireless networks," *Wireless Personal Commun.*, vol. 34, no. 1-2, pp. 45-66, July 2005.
[6] F. Cuomo, C. Martello, A. Baiocchi, and C. Fabrizio, "Radio resource sharing for ad hoc networking with UWB," *IEEE J. Sel. Areas Commun.*, vol. 20, no. 9, pp. 1722-1732, Dec. 2002.
[7] G. Giancola, C. Martello, F. Cuomo, and M.-G. Di Benedetto, "Radio resource management in infrastructure-based and ad hoc UWB networks," *Wireless Commun. and Mobile Comput.*, vol. 5, no. 5, pp. 581-597, 2005.
[8] R. Jurdak, P. Baldi, C. V. Lopes, "U-MAC: a proactive and adaptive UWB medium access control protocol," *Wireless Commun. and Mobile Comput.*, vol. 5, no. 5, pp. 551-566, 2005.
[9] H. Wymeersch and M. Moeneclaey, "ML rate detection for multi-rate TH-UWB impulse radio," in *Proc. IEEE ICU 2005*, pp. 391-395.
[10] M. Nasiri-Kenari and M. G. Shayesteh, "Performance analysis and comparison of different multirate TH-UWB systems: uncoded and coded schemes," *IEE Proc.-Commun.*, vol. 152, no. 6, pp. 833-844, Dec. 2005.