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Non-cooperative Spectrum Access – The Dedicated vs. Free Spectrum Choice

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Abstract

We consider a dynamic spectrum access system in which Secondary Users (SUs) choose to either acquire *dedicated spectrum* or to use *spectrum-holes* (white spaces) which belong to Primary Users (PUs). The trade-off incorporated in this decision is between immediate yet costly transmission and free but delayed transmission (a consequence of both the possible appearance of PUs and sharing the spectrum holes with multiple SUs). We first consider a system with a single PU band, in which the SU decisions are *fixed*. Employing queueing-theoretic methods, we obtain explicit expressions for the expected delays associated with using the PU band. Based on that, we then consider *self-interested* SUs and study the interaction between them as a *non-cooperative* game. We prove the existence and uniqueness of a symmetric Nash equilibrium, and characterize the equilibrium behavior explicitly. Using our equilibrium results, we show how to maximize revenue from renting dedicated bands to SUs and briefly discuss the extension of our model to multiple PUs. Finally, since spectrum sensing can be resource–consuming, we characterize the gains provided by this capability.

I. Introduction

This paper focuses on theoretical problems stemming from the decision process of users that can either participate in a Cognitive Radio Network (also known as Dynamic Spectrum Access Network) as Secondary Users or pay for temporary usage of a dedicated band. A Cognitive Radio (CR) was first defined by Mitola [2] as a radio that can adapt its transmitter parameters to the environment in which it operates. It is based on the concept of Software Defined Radio

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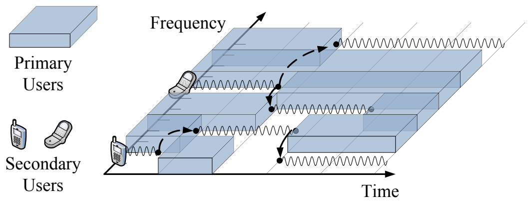

Fig. 1. An example illustrating Secondary Users (SUs) utilizing *white spaces* (also known as *spectrum holes*) that are not used by Primary Users (PUs).

(SDR) [3] that can alter parameters such as frequency band, transmission power, and modulation scheme through changes in software. According to the Federal Communications Commission (FCC), a large portion of the assigned spectrum is used only sporadically [4], [5]. Due to their adaptability and capability to utilize the wireless spectrum opportunistically, CRs are key enablers to efficient use of the spectrum. Hence, their potential has been recently identified by various policy [4], [6], research [7], standardization [8], [9], and commercial organizations.

Under the basic model of CR networks [10], Secondary Users (SUs) can use *white spaces* that are not used by the Primary Users (PUs) but must avoid interfering with active PUs (e.g., Fig. 1).[1] For example, the PUs and SUs can be viewed as public safety and commercial users, respectively, where the SUs must vacate the channel at very short notice. Another example is of PUs being TV broadcasters and SUs being commercial cellular operators using available TV bands [8]. Networks operating according to this model have distinct characteristics that pose numerous challenging theoretical and practical problems, of which many remain to be solved, despite extensive recent research (for a comprehensive review of previous work see [11], [12]).

Our work is motivated by a recent FCC ruling [6] that allows CR devices (SUs) to operate in TV bands white spaces. In addition to spectrum-sensing capability, these devices may include a geolocation capability and provisions to access a database that contains the PUs (e.g., TV stations) expected channel use. Given the geolocation capability, spectrum sensing is required in order to avoid interference to PU devices that are not registered in the database. The FCC will also certify CR devices that do not include the geolocation and database access capabilities, and rely solely on sensing.

The operation model described in [6] introduces a new set of theoretical problems at the

[1]PUs and SUs are also referred to as Licensed and Opportunistic Users, respectively.

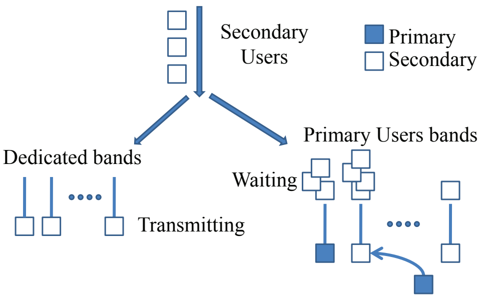

Fig. 2. An illustration of the decision process of the SUs and the arrival process of the PUs.

intersection of queueing theory, game theory, and control theory. In particular, we are interested in noncooperative SUs that have a spectrum sensing capability and can sense the PU band (e.g., *Wireless Internet Service Providers* - WISPs). These SUs can rent a licensed dedicated band, for a certain cost (we refer to such band as a *dedicated band*). Alternatively, they can use a band that is originally allocated to a PU (we refer to it as a *PU band*), for free. We assume that the SUs are service providers (i.e., they serve many users) that aggregate several connections/calls/packets to jobs that can be served over each of these band types. We do not focus on specific packets sent by specific users but rather on jobs that may be composed of several packets.

We study the decision process of the SUs which is illustrated in Fig. 2. An SU that has a job to serve can choose to use either one of the PU bands, or a dedicated band. When an SU selects a PU band, the band can be reclaimed by a PU and it is also *shared with other SUs that selected the same band*. Hence, the decision process of the SUs is affected by the tradeoff between the cost of acquiring a dedicated band and using a free PU band, which is prone to delays.

Our first step towards understanding the SU decision process is to consider a system with a *single PU band*. For such a system, we first study the delay performance when the SU decisions are *fixed*. To that end, we develop a queueing model based on a server with breakdowns [13]–[15], where the PU band is the server and the return of the PU is modeled as a breakdown. We assume that upon selection of the PU band, the SU joins a queue of SUs waiting to use that band. This corresponds to a server with breakdowns model in which the arrival rates depend on the server's status. To the best of our knowledge, this particular queuing model has not been rigorously considered in past literature.

We note that since managing a queue requires centralized control (which may not be feasible in a real system), a queue will most likely be replaced by a distributed MAC protocol (e.g., IEEE 802.22 [8]). In our analysis, we use the queue to represent the *congestion effect* incurred when a few SUs wish to use the same PU band.[2] We note that a number of recent works in the area of cognitive radio used "virtual" queues as a plausible model to capture SU congestion effects [19]–[21].

Based on the queueing analysis for fixed SU policies, we then study the SU decision process in a system with a single PU band. We prove the *existence* and *uniqueness* of a symmetric Nash equilibrium and fully characterize the equilibrium behavior for the SU decision strategies. Next, we apply our Nash equilibrium analysis to show how to maximize the revenue from renting dedicated bands to SUs that prefer not to use the PU band. Such information may be used by a spectrum broker that provides dedicated bands for short periods of time.

A system with SUs and PUs was modeled in [19] using the priority queueing model. While for a single PU band the two models are somewhat similar, we find the server-breakdown queueing model more natural and more appropriate for the *multi-band case*. In particular, the system can be modeled so that each PU band is a server prone to breakdowns (i.e., return of the PU) and there are queues (or a single queue) of SUs that can be served by any of the available PU bands. On the other hand, under the classical priority queueing model, there is a single queue of high priority users (PUs) and each of them can be served by any of the servers (PU bands). This does not comply with the operation model in which each PU has a dedicated band. Based on this observation, we can extend the model to the case with multiple PU bands. We also point out that the band pricing analysis extends to some special multi-band cases.

Finally, we study the *effect of the spectrum sensing capability of the SUs, on their average total cost* (namely, the delay cost plus monetary cost). It is of theoretical interest to understand the gain provided by spectrum sensing, since using this functionality (especially across multiple bands) requires some additional resources. We show that in some cases removing the sensing capabilities increases the SUs' cost and in other cases it has no effect. Hence, the Braess' paradox [22] of classical game theory, wherein the addition of resources to a system can actually worsen

[2]Since we are primarily interested in gaining insight into the SU band selection dynamics and for the sake of exposition, we do not focus on the contention for a channel (contention between similar users has been extensively studied [16]–[18]).

the overall system performance, does not occur in our system. However, there are cases in which the addition of a resource (sensing capability) does not improve the individual cost.

Unlike most of the previous work in the area of dynamic spectrum access, we utilize methods developed for decision making and the corresponding equilibrium analysis in queueing systems (see Haviv and Hassin [23] for a survey). Within that discipline, the novelty of the paper is in the analysis of the unobservable queue case and in examining the consequences of the dedicated band prices on the (non-cooperative) behavior of SUs and (for more details, see Section II). For tractability, we assume that the inter-arrival times and the service times are exponentially distributed. Relaxing some of these assumptions is a subject for future work.

To conclude, the main contribution of this paper is twofold. First, we develop a novel approach for the analysis of a dynamic spectrum access system. It combines the tools of game theory and queueing theory to provide insights into the SUs decision process as well as the spectrum pricing mechanisms used by the spectrum broker. Second, motivated by dynamic spectrum access systems, we provide novel results for the queueing theoretical problem of a server with breakdowns in which the arrival rates depend on the server's status.

This paper is organized as follows. In Section II we discuss related work and in Section III we present the model. We study the equilibrium of the SUs interactions in Section IV. In Section V we consider the problem of pricing the dedicated spectrum and briefly discuss the extension to the multi-band case. Finally, we examine in Section VI whether the SUs benefit from their sensing abilities. We conclude and discuss future research directions in Section VII.

II. Related Work

The extensive previous work in the area of CR as well as Cognitive Radio Network architectures, key enabling technologies, and recent developments have been summarized in a number of special issues and review papers (e.g., [11], [12]). In this section, we briefly review previous work which is most closely related to our model.

A practical MAC protocol (IEEE 802.22) that takes the CR characteristics into account has been studied in [24]. [25]–[28] used techniques from the area of Partially Observable Markov Decision Processes (POMDP) to model the behavior of PUs and SUs. Based on these techniques, decentralized protocols have been proposed. In [29], probabilistic methods have been used to evaluate the performance of PUs and SUs under different operation models. In [19]–[21], systems with SUs and PUs were modeled using priority queueing techniques. As mentioned above, we find the server-breakdown model more appropriate for modeling such a system.

Several papers used game theoretical notions to compare the cooperative and non-cooperative behavior of spectrum sensing and sharing [30]–[35]. In particular, [33] proposes a scheme in which users exchange "price" signals, that indicate the negative effect of interference at the receivers, [32], [34] deal with cases in which operators compete for customers, [35] studies a dynamic spectrum leasing paradigm, and [30] proposes a distributed approach, where devices negotiate local channel assignments aiming for a global optimum.

Unlike most of the previous work, we utilize methods developed for decision making in queueing systems [23]. Following [36], [37], extensive effort has been dedicated in the past decades to studying the effect of *pricing* on equilibrium performance. Our contribution is in analyzing the effect of the dedicated band pricing on the (non-cooperative) behavior of SUs. Recently, [38], [39] studied the decision process of customers who may join a server that can go on *vacation*. Under that model, the server stops serving customers for some (stochastically distributed) period, whenever it becomes idle. Our model, which corresponds to a server with breakdowns, is significantly different as the "server" (band) may stop serving customers (SUs) even when there are customers (SUs) waiting. In [40] decisions for the server with breakdowns model under the *observable* queue case (i.e., customers observe the queue size when making a decision) were studied. We, on the other hand, study the *unobservable* case which better approximates a distributed MAC employed by the SUs.

III. The Model

A. Preliminaries

We start by defining the model for a system with a single Primary User band and multiple Secondary Users that may wish to share that band. Our baseline model consists of a single PU who owns a spectrum band of some fixed bandwidth. The use of the PU band by the PU occurs intermittently, in the form of *sojourns*. We assume that the PU sojourn times (i.e., the amount of time that the PU uses its band at a stretch) are random and exponentially distributed with mean $1/\eta$. Moreover, the amount of time that elapses between the end of a sojourn, and the commencement of the next sojourn is also exponential with parameter ξ, and is independent of the sojourn times.

The SUs arrive to the network according to a Poisson process with rate λ. Each SU requires service for a random amount of time (exponential with parameter μ) in order to complete service. These SU 'job sizes' are assumed to be independent of the SU arrivals, and of the PU sojourns.

Upon arrival, each SU has to make a *spectrum decision*. That is, it has to decide between acquiring a *dedicated band* for a price, and using the *PU band* for free. If an SU chooses to acquire a dedicated band, it pays a fixed price $\tilde{C}$.[3] For simplicity, we assume that the dedicated band and the PU band have the same bandwidth. Hence, the SU's service times are exponential with parameter μ in either case. If an SU chooses to use the PU band, it joins a virtual queue of SUs who have chosen to use the PU band. This queue is used in order to model the delay incurred when a few SUs wish to use the same PU band.[4]

The SUs can sense the PU band and learn whether the PU is present.[5] Yet, the SU does not know how many other SUs are presently attempting to use the PU band, and must make its decision only on the basis of statistical information. This models the case in which SUs try to ditributedly access a channel (e.g., by a MAC protocol) and are not centrally managed.

The average cost incurred by a secondary user consists of two components: (i) the price of the dedicated band $\tilde{C}$, and (ii) an average delay cost. Let α be the delay cost per unit time (i.e., α represents the delay vs. monetary cost tradeoff of the SUs). The expected cost when acquiring dedicated spectrum is thus given by

$$J_B = \tilde{C} + \frac{\alpha}{\mu} = C. \tag{1}$$

We will refer to $\tilde{C}$ as the *dedicated band price*, and to C as the *total dedicated band cost*.

The expected cost of using the PU band consists purely of a delay cost. Specifically, it is given by α times the expected delay faced by the SU. This expected delay depends on the presence or absence of the PU, as discussed in the next section.

B. SU Strategies

Since the SUs can sense the presence or absence of the PU, they can compute the expected delay cost *conditioned* on their sensing outcome. In particular, SUs which sense the PU to be

[3] We assume that there is no lack of dedicated bands, so that a user who is willing pay for a dedicated band can get it.

[4] In a real system, the contention for a channel may be realized by a distributed MAC protocol rather than by a queue.

[5] We assume that SUs can distinguish between a PU and an SU using, for example, the packet header or activity pattern.

present see a different conditional delay, and can therefore adopt a different strategy from those which sense the PU to be absent. In this work, we consider strategies that are described by a pair of fractions (p, q), where p is the probability that an SU decides to use the PU band, given that the PU is absent (thus, with probability $1 - p$ it acquires dedicated spectrum), and q is the probability that an SU decides to use the PU band, given that the PU is present (thus, with probability $1 - q$ it acquires dedicated spectrum).

C. Nash Equilibrium

The classic notion of a Nash equilibrium stands for an operating point (a collection of strategies) where no user can improve its cost by unilaterally deviating from its current strategy. We wish to characterize the equilibrium points for the simple class of strategies outlined above.

For a strategy (p, q), let $T_A(p, q)$ denote the conditional delay experienced by an SU that arrives when the PU band is available and $T_O(p, q)$ be the conditional delay experienced by an SU that arrives when the PU band is occupied.[6] The corresponding delay costs are given by $J_A(p, q) = \alpha T_A(p, q)$, and $J_O(p, q) = \alpha T_O(p, q)$.

In this paper, we will restrict attention to *symmetric* Nash equilibria, as a common solution approach in the research of equilibrium behavior in queuing systems [23]. While asymmetric equilibria may exist, their study remains beyond the scope of the present paper. It can be easily seen that a pair (p, q) is a (symmetric) Nash equilibrium, if and only if one relation from each of (2) and (3) holds.

$$\text{(i)} J_A(p,q) \leq C, \ \& \ p = 1; \quad \text{(ii)} J_A(p,q) = C, \ \& \ 0 < p < 1; \quad \text{(iii)} J_A(p,q) \geq C, \ \& \ p = 0 \quad (2)$$

$$\text{(i)} J_O(p,q) \leq C, \& \ q = 1; \quad \text{(ii)} J_O(p,q) = C, \ \& \ 0 < q < 1; \quad \text{(iii)} J_O(p,q) \geq C, \ \& \ q = 0. \quad (3)$$

To avoid a trivial solution, we make the following assumption throughout the paper.

Assumption 1: The total dedicated band cost satisfies the following inequalities: $J_A(0, 0) < C$, and $J_O(1, 1) > C$.

Above, $J_A(0, 0)$ should be interpreted as the delay cost incurred, if a specific SU were to join the cognitive queue when the PU is absent, given that no other SU chooses to join the queue.

[6] T_A and T_O depend on both p and q, since these delays are a function of the previous SU arrivals that have occurred.

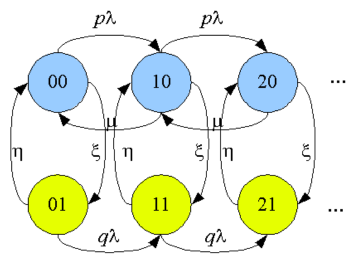

Fig. 3. Queue occupancy Markov process.

IV. Equilibrium Analysis

We now analytically characterize equilibrium behavior of the SUs. As a building block, we obtain in Section IV-A the conditional delay expressions T_A and T_O for a *given* strategy (p, q). Using the delay analysis, we provide several basic properties of the equilibrium in Section IV-B. These are then used in Section IV-C to fully characterize the equilibrium behavior.

A. Conditional Delays

We develop explicit formulas for the conditional delays T_A and T_O for given values of the probabilities p and q. We view the arrival of a PU as a server breakdown. That is, when a PU arrival occurs, the SU being served at that time is preempted, and service resumes after an exponentially distributed interval of mean duration $1/\eta$. Since the service time distribution of the SUs is memoryless, the remaining service time of a preempted SU is still exponential with parameter μ. While delay analysis of exponential servers under breakdown has been studied extensively [13]–[15], our analysis is significantly more involved because the instantaneous arrival rate of SUs to the queue is a function of the presence or absence of the PU.

Fig. 3 depicts the Markov process corresponding to the system evolution. In the chain, the state $i0$ denotes the absence of a PU, and the presence of i SUs, where $i = 0, 1, \ldots,$ and $i1$ denotes the presence of a PU and i SUs. Note that the arrival process of SUs is Poisson of rate $p\lambda$ when the PU is present, and Poisson of rate $q\lambda$ when the PU is not present. This follows from the splitting property of Poisson processes. Further, SUs get served at rate μ when the PU is not present, and do not get served when the PU is present.

The steady state probability of a PU being absent can be easily shown to be $\eta/(\eta+\xi)$. The Markov process is positive recurrent if the average arrival rate is less than the average service rate, i.e., $(p\eta\lambda + q\xi\lambda)/(\eta+\xi) < \mu\eta/(\eta+\xi)$. For simplicity, we assume that the system is stable for *all* values of p and q, which implies $\lambda < \mu\eta/(\eta+\xi)$. Under the above conditions, we next obtain explicit formulas for T_A and T_O, which will be used for the equilibrium characterization.

Theorem 1: Let p and q be the probabilities of an SU committing to take the PU band, in case that the PU band is available and in case that it is occupied, respectively. The respective conditional delays are given by

$$T_A(p,q) = \frac{\eta+\xi}{\mu\eta - \eta p\lambda - q\lambda\xi}\left(1 + \frac{q^2\lambda^2\xi}{\mu\eta^2}\right) \tag{4}$$

and

$$T_O(p,q) = \frac{\eta+\xi+\mu-(p-q)\lambda - \frac{pq\lambda^2(\eta+\xi)}{\mu\eta}}{\mu\eta - \eta p\lambda - q\lambda\xi}. \tag{5}$$

Proof: The proof follows through the following three steps. We first obtain the steady-state probabilities for the Markov chain and then derive the conditional occupancy of SUs. Next, we compute the average time spent at the 'head of line' of the queue by an SU. We combine the above two steps by invoking an 'Arrivals See Time Averages' property in order to obtain the required expressions.

Step 1: Steady-State Probabilities and conditional occupancy.

Let us first balance the rate of increase in the total number of SUs, with the rate of decrease. This gives

$$p_{i+1,0}\mu = p\lambda p_{i,0} + q\lambda p_{i,1}, i \geq 0 \tag{6}$$

Summing the above equation over all $i \geq 0$, and noting the relations $\sum_{i\geq 0} p_{i,0} = \frac{\eta}{\eta+\xi}$ and $\sum_{i\geq 0} p_{i,0} = \frac{\xi}{\eta+\xi}$, we obtain

$$p_{0,0} = \frac{\eta}{\eta+\xi}(1 - \frac{\lambda}{\mu}(p + q\frac{\xi}{\eta})).$$

Next, the following balance equations for the steady-state probabilities can be deduced from Fig. 3.

$$p_{0,1} = \frac{\xi}{\eta + q\lambda} p_{0,0}$$

$$p_{1,0} = p_{01}\frac{q\lambda}{\mu} + p_{0,0}\frac{p\lambda}{\mu}$$

$$p_{i,0}(p\lambda + \mu + \xi) = p_{i-1,0}p\lambda + p_{i+1,0}p\mu + p_{i,1}\eta,\ \ i \geq 1 \tag{7}$$

$$p_{i,1}(q\lambda + \eta) = p_{i-1,1}q\lambda + p_{i,0}\xi,\ \ i \geq 1 \tag{8}$$

Since $p_{0,0}, p_{1,0}$ and $p_{0,1}$ are now known, we can in principle recursively obtain all the steady state probabilities using (7) and (8). However, in order to obtain closed form formulas, we treat (7,8) as a system of linear coupled difference equations. We can eliminate $p_{m,1}$ using the coupled difference equations[7] to yield a homogeneous third order difference equation as shown in (9) for $m \geq 2$.

$$p_{i+1,0}\mu(q\lambda+\eta)-p_{i0}((p\lambda+\mu+\xi)(q\lambda+\eta)+\mu q\lambda-\eta\xi)+p_{i-1,0}(2pq\lambda^2+p\lambda\eta+q\lambda\mu+\xi q\lambda)-p_{i-2,0}pq\lambda^2 = 0 \tag{9}$$

Standard methods exist to solve such difference equations. In particular, $p_{m,0}$ can be shown to have the form

$$p_{m,0} = A\beta_+^m + B\beta_-^m + C\beta_1^m,$$

where $\beta_+, \beta_-, \beta_1$ are zeros of the characteristic polynomial corresponding to the difference equation in (9). The characteristic polynomial is given by

$$x^3\mu(q\lambda + \eta) - x^2((p\lambda + \mu + \xi)(q\lambda + \eta) + \mu q\lambda - \eta\xi) + x(2pq\lambda^2 + p\lambda\eta + q\lambda\mu + \xi q\lambda) - pq\lambda^2.$$

$\beta_1 = 1$ is clearly a zero of the polynomial. The other two roots are given by

$$\beta_\pm = \frac{a \pm \Delta}{2b} \tag{10}$$

where $\Delta = \sqrt{a^2 - 4bp\lambda}$, $a = p\lambda + \eta\frac{p}{q} + \xi + \mu$, and $b = \frac{\mu(q\lambda+\eta)}{q\lambda}$. Then, $p_{m,0}$ is given

$$p_{m,0} = (C_+\beta_+^{m-1} + C_-\beta_-^{m-1})p_{1,0},\ \ m \geq 1. \tag{11}$$

We can solve for C_+ and C_- by noting that $C_+ + C_- = 1$, and $\sum_{m=0}^{\infty} p_{m,0} = \frac{\eta}{\eta+\xi}$. This yields

[7]Write equation (7) with $m-1$ replacing m, and call it (12a). Now subtract $q\lambda$ times (12a) from $q\lambda + \eta$ times (7), and use (8) to eliminate $p_{m,1}$.

$$C_+ = \frac{b}{\Delta}\left(\beta_+ - \frac{pq\lambda}{p\eta + pq\lambda + q\xi}\right),$$

and

$$C_- = \frac{b}{\Delta}\left(\frac{pq\lambda}{p\eta + pq\lambda + q\xi} - \beta_-\right).$$

Next, $p_{m,1}$ can be solved for from (8) by substituting for $p_{i,0}$ which are now known. This yields (12).

$$p_{m,1} = \frac{\xi}{q\lambda+\eta}\left(\frac{q\lambda}{q\lambda+\eta}\right)^m p_{00} + p_{10}\left\{\frac{\xi C_-}{\beta_-(q\lambda+\eta)-q\lambda}\left[\beta_-^m - \left(\frac{q\lambda}{q\lambda+\eta}\right)^m\right] + \frac{\xi C_+}{\beta_+(q\lambda+\eta)-q\lambda}\left[\beta_+^m - \left(\frac{q\lambda}{q\lambda+\eta}\right)^m\right]\right\}. \quad (12)$$

Next, let us denote by $\overline{N}_A$ ($\overline{N}_O$) the average SU occupancy when the PU is absent (present). Thus,

$$\overline{N}_A = \left(1+\frac{\xi}{\eta}\right)\sum_{m=1}^{\infty} m p_{m0}, \quad (13)$$

$$\overline{N}_O = \left(1+\frac{\eta}{\xi}\right)\sum_{m=1}^{\infty} m p_{m1}. \quad (14)$$

Since the steady state probabilities in (13) and (14) are known, we can obtain $\overline{N}_A$ and $\overline{N}_O$ in closed form, as shown in (15) and (16).

$$\overline{N}_A = \left(1+\frac{\xi}{\eta}\right)p_{10}\left[\frac{C_+}{(1-\beta_+)^2} + \frac{C_-}{(1-\beta_-)^2}\right], \quad (15)$$

$$\overline{N}_O = \left(1+\frac{\eta}{\xi}\right)\left(p_{00}\xi\frac{q\lambda}{\eta^2} + \frac{p_{10}\xi C_-}{(\beta_-(q\lambda+\eta)-q\lambda)}\left[\frac{\beta_-}{(1-\beta_-)^2} - q\lambda\frac{q\lambda+\eta}{\eta^2}\right] + \frac{p_{10}\xi C_+}{(\beta_+(q\lambda+\eta)-q\lambda)}\left[\frac{\beta_+}{(1-\beta_+)^2} - q\lambda\frac{q\lambda+\eta}{\eta^2}\right]\right); \quad (16)$$

Step 2: Head-of-line delay. Let τ_{HoL} denote the average time spent at the head of line of the queue by an SU. This time has two components: the time for service, which is exponential with mean $1/\mu$, plus the time for which the server is broken down (because of a PU arrival). Once an SU enters service, it completes service before being preempted by a PU with probability $\mu/(\mu+\xi)$. If it is preempted by a PU, it stays at the head-of-line for a mean duration of $1/\eta$, after which the service is resumed. Since the distribution of the SU service time is memoryless, the following recursion is straightforward:

$$\tau_{HoL} = \begin{cases} \frac{1}{\mu+\xi} & w.p.\ \frac{\mu}{\mu+\xi} \\ \frac{1}{\mu+\xi} + \frac{1}{\eta} + \tau_{HoL} & w.p.\ \frac{\xi}{\mu+\xi} \end{cases}$$

Thus, we get

$$\tau_{HoL} = \frac{1}{\mu}\left(1 + \frac{\xi}{\eta}\right). \tag{17}$$

*Step 3: Conditional delays seen upon arrival.*Let $\hat{N}_O$ and $\hat{N}_A$ respectively denote the average queue occupancy seen by an SU, upon arriving to an occupied or available queue, respectively. Since each packet spends an average duration of τ_{HoL} at the head-of-line, we have the following relations for the conditional delays T_A and T_O:

$$T_A = (1 + \hat{N}_A)\tau_{HoL} \tag{18}$$

$$T_O = \frac{1}{\eta} + (1 + \hat{N}_O)\tau_{HoL} \tag{19}$$

We comment that the average occupancy seen by an arriving SU need not, in general, equal the time average occupancy seen by an external observer. However, we argue in the appendix that the 'Arrivals See Time Averages' (ASTA) property holds, once we condition on the presence or absence of the PU. Thus, $\overline{N}_A = \hat{N}_A$ and $\overline{N}_O = \hat{N}_O$. As a result, the expressions for the conditional delays read

$$T_A = (1 + \overline{N}_A)\tau_{HoL}, \tag{20}$$

$$T_O = \frac{1}{\eta} + (1 + \overline{N}_O)\tau_{HoL}, \tag{21}$$

where $\overline{N}_A, \overline{N}_O$ and τ_{HoL} are given in (15), (16), and (17) respectively. Substituting and simplifying gives (4) and (5). □

As expected, the average delay experienced by an SU that arrives when the server is occupied is strictly greater than that experienced by an SU that arrives when the server is available.

Proposition 2: For any p, q we have $T_O(p,q) > T_A(p,q)$.

Proof: From (20) and (21), the result would follow, if $\overline{N}_O \geq \overline{N}_A$. Since the event of a PU arrival is a memoryless event, it is clear that the average occupancy just before a PU arrival is equal to $\overline{N}_A$. Thus, the average SU occupancy just after the PU arrival is also $\overline{N}_A$. Since the SUs get no service after the PU arrival, the average SU occupancy when the PU is present ($\overline{N}_O$) cannot be smaller than the occupancy just after the PU arrival. Thus, $\overline{N}_O \geq \overline{N}_A$. □

B. Basic Equilibrium Properties

We prove in this subsection that the Nash equilibrium point exists and is unique. Along the way, we describe additional properties of the equilibrium. We start by stating that an equilibrium point always exists.

Proposition 3: There always exists a Nash equilibrium.

Proof: Let us consider three possible cost ranges, and show the existence of equilibrium in each case: Case (i) $J_A(1,0) \leq C, J_0(1,0) \geq C$. Noting (2)–(3), $(p,q) = (1,0)$ is a Nash equilibrium for this case. Case (ii) $J_A(1,0) > C$. Recall that $J_A(0,0) < C$ by assumption. Then by continuity of the delay function, it follows that there exists $p < 1$ such that $J_A(p,0) = C$ (intermediate-value theorem); furthermore, $J_O(p,0) > C$ by Proposition 2. In view of (2)–(3), the last two assertions immediately imply that $(p,0)$ is a Nash equilibrium. Case (iii) $J_O(1,0) < C$. Recall that $J_O(1,1) > C$ by assumption. Then by continuity of the delay function, it follows that there exists $q < 1$ such that $J_O(1,q) = C$; furthermore, $J_A(1,q) < C$ by Proposition 2. In view of (2)–(3), the last two assertions immediately imply that $(1,q)$ is a Nash equilibrium. Thus, there always exists an equilibrium point. □

We next provide a basic characterization of the range of equilibrium probabilities.

Proposition 4: Suppose that the pair (p,q) is a Nash equilibrium. Then, (i) $0 < p < 1 \implies q = 0$. (ii) $0 < q < 1 \implies p = 1$.

Proof: Using (2), we see that the condition $0 < p < 1$ implies $C = J_A(p,q)$. Next, proposition 2 implies $J_O(p,q) > J_A(p,q) = C$. Finally, using (3), we conclude that $q = 0$. Part (ii) also follows along similar lines. □

Note that the above proposition, together with Assumption 1, imply that $p > q$ in any equilibrium, as might have been expected. By using this proposition, we can now establish the uniqueness of the equilibrium point.

Proposition 5: The Nash equilibrium point is unique.

Proof: The proofs follows from the following auxiliary lemma.

Lemma 1: Let (p_1, q_1) and (p_2, q_2) be two distinct Nash equilibria. Then, (i) $p_1 > p_2 \implies q_1 \geq q_2$. (ii) $q_1 > q_2 \implies p_1 \geq p_2$.

Proof: (i) Assume to get a contradiction that $q_2 > q_1$, hence, $q_2 > 0$. If $q_2 = 1$ then $p_2 = 1$, which cannot be an equilibrium by Assumption 1; otherwise, $0 < q_2 < 1$, which by Proposition 4(ii) suggests that $p_2 = 1$, a contradiction. (ii) Assume by contradiction that $p_1 < p_2$, hence $p_1 < 1$. If $p_1 = 0$ then $q_1 = 0$, which cannot be an equilibrium by Assumption 1; otherwise, $0 < p_1 < 1$, which by Proposition 4(i) suggests that $q_1 = 0$, a contradiction. □

It follows by the above lemma that if there exist two different equilibria (p_1, q_1), (p_2, q_2), then (without loss of generality) (a) $p_1 > p_2$, $q_1 \geq q_2$ or/and (b) $p_1 \geq p_2$, $q_1 > q_2$. We can show that both (a) and (b) lead to a contradiction. Indeed, (a) implies that $C \geq J_A(p_1, q_1) > J_A(p_2, q_2) \geq C$, (where the first and third inequality follow from (2), and the second since the congestion in equilibrium 1 is strictly higher than in equilibrium two), which is a contradiction. Similarly, assuming (b), we obtain the following contradicting inequality $C \geq J_O(p_1, q_1) > J_O(p_2, q_2) \geq C$. We conclude that we cannot have multiple equilibria, hence the Nash equilibrium is unique. □

C. Characterization of the Nash Equilibrium

Next, we characterize the equilibrium behavior of the SUs for a given cost C. Proposition 4, together with assumption 1 implies that a Nash equilibrium pair (p, q) can only have one of the following three forms: (a) $(1, q), 0 < q < 1$ (b) $(1, 0)$, and (c) $(p, 0), 0 < p < 1$. In the following theorem, we identify three ranges of the total dedicated band cost for which the above three forms of equilibria are observed, and explicitly obtain the equilibrium probabilities as a function of C.

Theorem 6: The equilibrium probabilities p and q can be characterized as a function of the cost C as follows:

(i) If $J_O(1, 0) < C < J_O(1, 1)$, the Nash equilibrium pair is $(1, q(C))$, where

$$q(C) = \frac{\mu\eta\left((\eta\frac{C}{\alpha} - 1)(\mu - \lambda) - (\eta + \xi)\right)}{\lambda(\frac{C}{\alpha}\eta\mu\xi + \eta\mu - \lambda(\eta + \xi))}, \tag{22}$$

In words, a fraction $q(C)$ of the users who arrive to find the free spectrum occupied, still join the queue, while all the users who find the free spectrum available, join the free spectrum.

(ii) If $J_A(1,0) \leq C \leq J_O(1,0)$, the equilibrium pair is (1,0). That is, all SUs take the PU band if available, and no SU takes the PU band if it is occupied.

(iii) If $J_A(0,0) < C < J_A(1,0)$, the equilibrium pair is $(p(C), 0)$, with

$$p(C) = \frac{\mu}{\lambda} - \alpha \frac{1 + \frac{\xi}{\eta}}{C\lambda} \tag{23}$$

In this case, a fraction $p(C)$ of the users who find the server available join the free spectrum, while all the users who arrive to find the server occupied acquire dedicated spectrum.

Proof: If C satisfies case (ii), we see that the equilibrium conditions (2,3) are satisfied with $p = 1$ and $q = 0$. Next suppose that C satisfies case (i). Consider the function $J_O(1, q),\ q \in (0, 1)$ which, as we might expect, is continuous and increasing in q. As a result, there exists a unique $0 < q(C) < 1$ such that $J_O(1, q) = C$. Indeed, this equation can be explicitly inverted to yield $q(C)$ in (22). Thus, the equilibrium condition (3) is satisfied with equality. Further, since $q < 1$, proposition (4) implies $p = 1$, and it follows that $(1, q(C)$ is an equilibrium pair. Case (iii) follows along similar lines. □

Using the relation between C and $\tilde{C}$, (1), we can also obtain the equilibrium probabilities in terms of the band price $\tilde{C}$. With some notation abuse, (1) and (22) together yield $q(\tilde{C}) = (K\tilde{C} - L)/(A\tilde{C} + B)$, with $K = \mu\eta^2(\mu - \lambda)/\alpha,\ L = \mu\eta(\mu - \lambda + \xi + \lambda\eta/\mu),\ A = \lambda\xi\eta\mu/\alpha$, and $B = \lambda\eta\xi + \mu\lambda\eta - \lambda^2(\eta + \xi)$. Similarly, from (1) and (23), $p(\tilde{C}) = \frac{\mu}{\lambda} - \alpha\frac{1+\xi/\eta}{(\tilde{C}+\alpha/\mu)\lambda}$. Fig. 4 shows a plot of the probabilities p and q as a function of the band price $\tilde{C}$ for a particular system.

V. Revenue Maximizing Pricing

Since the SU strategy depends on the cost of a dedicated band, a service provider may wish to price the dedicated bands so as to maximize its revenue. We make here the assumption that the dedicated spectrum is owned by a single provider (a monopoly), who may unilaterally adjust the price $\tilde{C}$. The natural tradeoff the monopoly faces is between obtaining more revenue per customer and attracting more customers to the dedicated spectrum by reducing the price per customer. In Section V-A we provide full characterization of the single-band revenue maximizing price. In Section V-B we discuss the extension of the analysis to the case where the monopolist owns multiple bands.

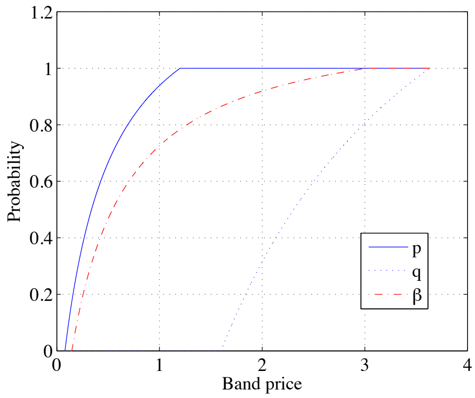

Fig. 4. Probability of committing to the PU band, as a function of the band price. The system parameters are $\mu = 10,\ \lambda = 7,\ \eta = 10,\ \xi = 2$, and $\alpha = 4$.

A. The Single-Band Case

Fig. 5 depicts the (equilibrium) total revenue as a function of the price $\tilde{C}$ for a given game instance. Note that the obtained function is neither concave nor convex, which might indicate that the optimal price can be solved for only numerically. However, we show below that the optimal price can be obtained very efficiently, requiring the revenue comparison under a maximum of only four alternatives, each of which given in a closed-form formula. This appealing result is formalized in the next theorem.

Theorem 7: For any given set of system parameters, consider the following four band prices: $\tilde{C}_1^* = \frac{1}{A}\sqrt{\frac{B(KB+AL)}{K-A}} - \frac{B}{A}$, (where $K = \mu\eta^2(\mu-\lambda)/\alpha,\ L = \mu\eta(\mu-\lambda+\xi+\lambda\eta/\mu),\ A = \lambda\xi\eta\mu/\alpha$ and $B = \lambda\eta\xi + \mu\lambda\eta - \lambda^2(\eta+\xi)$); $\tilde{C}_2^* = J_O(1,0) - \alpha/\mu$; $\tilde{C}_3^* = \alpha\sqrt{\frac{\eta+\xi}{\mu(\mu\eta-\lambda(\eta+\xi))}} - \alpha/\mu$; and $\tilde{C}_4^* = \alpha\xi/(\mu\eta)$. Define $\tilde{C}_2^*$ and $\tilde{C}_4^*$ to be candidate prices. Further, $\tilde{C}_1^*$ is a candidate price if $J_O(1,0) < \tilde{C}_1^* + \alpha/\mu < J_O(1,1)$, and $\tilde{C}_3^*$ is a candidate price if $J_A(0,0) < \tilde{C}_3^* + \alpha/\mu < J_A(1,0)$. Then, the globally optimal pricing policy is an *index policy*, which compares the revenues generated under each of the candidate prices, of which there are at most four.

The proof follows by separately considering each of the three cost subregions given in Theorem 6, as summarized in the next three lemmas.

Lemma 2: In the price range $J_O(1,0) < \tilde{C} + \alpha/\mu < J_O(1,1)$, the band price that maximizes the average revenue earned from the dedicated spectrum is given by $\tilde{C}_1^* = \frac{1}{A}\sqrt{\frac{B(KB+AL)}{K-A}} - \frac{B}{A}$, as long as $\tilde{C}_1^*$ lies in the above range. If $\tilde{C}_1^*$ does not lie in the range of interest, then the revenue generated is monotonically decreasing in the band price, and the optimal band price will be given by the next proposition.

Proof: In this case, a fraction $1 - q(\tilde{C})$ of the users acquire dedicated spectrum when the PU is present, while no SU acquires dedicated spectrum if the PU is absent. The average number of customers who acquire spectrum in a unit time is thus equal to $(1-q(\tilde{C}))\lambda\xi/(\xi+\eta)$. Since each customer pays a *monetary* cost[8] $\tilde{C}$, the rate of revenue generation is $\tilde{C}(1 - q(\tilde{C}))\lambda\xi/(\xi + \eta)$. Using basic Calculus, we can show that the rate of revenue generation is concave in $\tilde{C}$. The stationary point of the concave function, which is given by $\tilde{C}_1^*$, would be the optimal value for this range of band price, if it lies in the said range. If not, it can be shown that the revenue rate is monotone decreasing in the band price, and Lemma 3 would take over. □

Lemma 3: In the price range $J_A(1,0) \leq \tilde{C} + \alpha/\mu \leq J_O(1,0)$, the band price that maximizes the average revenue earned is given by $\tilde{C}_2^* = J_O(1,0) - \alpha/\mu$. In other words, it is optimal to price the spectrum at the highest value that leads to the equilibrium pair (1,0).

Proof: In this case, all the users who sense an available server take the PU band while the users who sense an occupied server acquire dedicated spectrum. Thus, it is clearly advantageous in terms of revenue to choose the highest band price allowed, which is equal to $J_O(1,0)$. □

Finally, we consider the optimal pricing corresponding to case (iii) of Theorem 6.

Lemma 4: In the cost range $J_A(0,0) \leq \tilde{C} + \alpha/\mu < J_A(1,0)$, the pricing that maximizes the average revenue earned from the dedicated spectrum is given by $\tilde{C}_3^* = \alpha\sqrt{\frac{\eta+\xi}{\mu(\mu\eta-\lambda(\eta+\xi))}} - \alpha/\mu$, as long as $\tilde{C}_3^*$ lies in the said range. If not, the optimum band price is $\tilde{C}_4^* = J_A(0,0) - \alpha/\mu = \alpha\xi/(\mu\eta)$.

Proof: In this range, the rate of revenue generation is given by $\tilde{C}\lambda\left(1 - \eta p(\tilde{C}/(\eta+\xi))\right)$, which is easily shown to be concave in $\tilde{C}$. The rest of the proof is akin to Lemma 2. □

Once the local optimum prices are determined according to the above lemmas, we can find the globally optimum price, by comparing the revenues under each locally optimum band price. This concludes the proof of Theorem 7. □

Returning to the example in Fig. 5, we see that the global optimum band price for the given game-instance is $\tilde{C}_3^* = 0.16$.

[8]Since we are interested in the revenue generated, the delay cost α/μ is not considered.

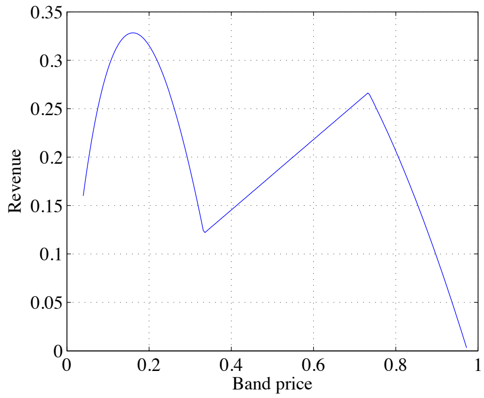

Fig. 5. An example of the revenue generated as a function of the band-price $\tilde{C}$ when the system parameters are $\mu = 10,\ \lambda = 4,\ \eta = 10,\ \xi = 1$, and $\alpha = 4$.

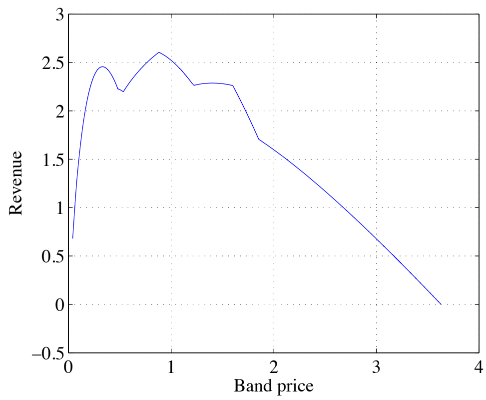

Fig. 6. An example of the revenue as a function of the band-price $\tilde{C}$ for 3 PU bands ($N = 3$). The system parameters are: $\mu_i = \mu = 10, \alpha_i = \alpha = 4, \lambda_1 = 7, \eta_1 = 10, \xi_1 = 1, \lambda_2 = 5, \eta_2 = 10, \xi_2 = 1, \lambda_3 = 5, \eta_3 = 6,$ and $\xi_3 = 1$.

B. Multiple Primary-User Bands

In this section, we consider the problem of choosing between free and dedicated spectrum, where several PU bands are available. Let N be the number of PUs in the system, each owning a different band. We denote by ξ_i and η_i the sojourn parameters of the PU in the ith PU band. We assume that each SU can sense only a small number of PU bands before making its spectrum decision. The spectrum decision is (as before) between committing to one of the sensed PU bands, based on the conditional delay estimates, or acquiring dedicated spectrum for a fixed unified price $\tilde{C}$.[9]

The study of the above model in its full generality (i.e., each SU may sense some subset of the available PU bands) naturally becomes an extremely difficult problem, even if one settles for numeric solutions. However, once additional assumptions are made, it may be possible to solve for the equilibrium point (and related aspects), either explicitly or numerically. We consider in this section a specific tractable scenario, and conclude by briefly mentioning an additional model which is subject of on-going investigation.

We consider next a simplified case of limited-sensing abilities, where each SU can sense only a *single* PU band before making its spectrum decision. This case is formally modeled as having an heterogenous SU population of N types, where all SUs of the ith type sense the ith PU band ($i = 1, \ldots, N$). The arrival rate of each type i is denoted λ_i, and it is assumed that all i-type SUs have the same service-time distribution (exponential with mean $1/\mu_i$, regardless whether they commit to their sensed band or acquire dedicated spectrum) and the same delay cost coefficient α_i.

A Nash equilibrium for the above defined system is characterized by $\{(p_i, q_i)\}_{i=1}^{N}$, where p_i is the probability that type i SUs commit to the ith band and q_i is the probability that they acquire dedicated spectrum. It can be easily seen that for a given price $\tilde{C}$, the equilibrium analysis decouples and can be solved separately for each PU band, by using the analysis of the preceding sections. Specifically, the conditional delays for each PU band can be derived using Theorem 1, with $\eta,\ \xi$, and λ replaced by $\eta_i,\ \xi_i$, and λ_i. Then, the equilibrium probabilities (p_i, q_i) can be obtained from Theorem 6.

The challenging issue which we next consider is how to set the optimal (revenue-maximizing) price $\tilde{C}$. Once the equilibrium probabilities are known for each i, the total revenue obtained from dedicated spectrum sales can be computed using the expression

$$R(\tilde{C}) = \sum_{i=1}^{N} \lambda \tilde{C} \frac{(1 - p_i)\eta_i + (1 - q_i)\xi_i}{\eta_i + \xi_i}.$$

Fig. 7 depicts the total revenue $R(\tilde{C})$ as a function of the price $\tilde{C}$ for a specific problem instance with three PU bands. The system parameters in this instance are: $\mu_i = \mu = 10, \alpha_i = \alpha = 4, \lambda_1 =$

[9]We assume that the SUs are risk-adversary, in the sense that they will not commit to a PU band which they have not sensed (and most likely are not aware of the statistical properties thereof).

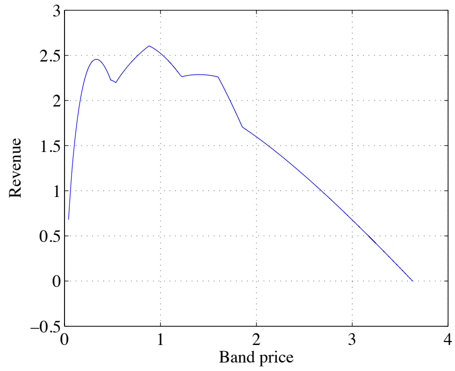

Fig. 7. Revenue as a function of the band-price $\tilde{C}$ for $N = 3$ PU bands

$7, \eta_1 = 10, \xi_1 = 1, \lambda_2 = 5, \eta_2 = 10, \xi_2 = 1, \lambda_3 = 5, \eta_3 = 6,$ and $\xi_3 = 1$. The optimal price is seen to be $\tilde{C}^* = 0.88$ monetary units. The equilibrium probabilities corresponding equilibrium probabilities are given by $(0.893, 0)$, $(1, 0)$, and $(1, 0)$ respectively. The figure demonstrates that even for a relatively small N, there are numerous price ranges to be considered, and the analytical optimization of band price is very cumbersome due to the intricate structure of the curve. Nonetheless, as the associated optimization problem is over a scalar variable $\tilde{C}$, one can always numerically solve for the optimal price in an efficient way, using standard search techniques (see, e.g., [41]).

We conclude this section by briefly mentioning an additional relevant model of a system with multiple PU bands. Assume that the number of PU bands N is relatively small and that each SU can sense *all* bands prior to its decision. Without further assumptions, we need an exponential number of probabilities to describe an equilibrium point, since there are 2^N possible subsets of PU bands that could be available at any time. It turns out that the state-space of this problem can be simplified significantly under some symmetry assumptions.

Suppose that the parameters of the PU sojourns, ξ and η, are are equal in all the bands. This assumption lends a certain symmetry to the problem. For instance the SU's decision to acquire dedicated spectrum only on *how many* PU bands are unoccupied and any instant, and not on which specific bands are occupied. Further, because of the symmetry, the conditional delay seen upon taking an occupied PU band is greater than the delay encountered at an available PU band. This implies that SUs never commit to an occupied PU band when at least one PU band

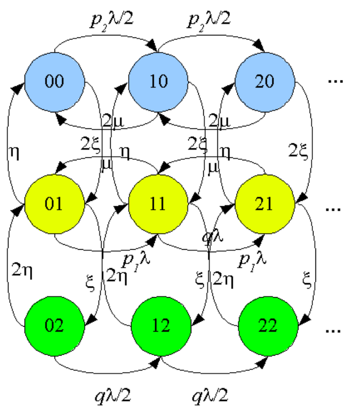

Fig. 8. Queue occupancy Markov process for $N = 2$ PU bands

is available. When k out of N PU bands are available, suppose that a fraction p_k of the SUs commit to the free spectrum, and the rest acquire dedicated spectrum. Further, suppose that the SU that takes the free spectrum commits to one of the available PU bands with equal probability.

Under these assumptions, we can reduce the state-space of the SU occupancy to $O(N)$. This leads to a Markov chain with $N + 1$ parallel-branches for the conditional-delay expressions, which can be simulated for obtaining steady state probabilities. These can in turn be used to study the equilibrium behavior. The case $N = 2$, albeit cumbersome, can be dealt with in closed form, along the lines of [14]. We note that for $N > 2$, there does not seem to be an analytic method for obtaining the equilibrium point. Specifically, the SU occupancy at any given PU band can be described in term of a Markov chain, which has $N + 1$ parallel branches, where the kth branch corresponds to $k - 1$ PUs being present, $k = 1, \dots, N + 1$. Fig. 8 shows such a Markov chain for $N = 2$ PU bands.

VI. The Effect of Sensing on the SU Cost

In this section, we study the hypothetical case in which SUs do not have the ability to sense the presence of PUs. In such a case, the SUs decide to use the free band or buy dedicated spectrum based on statistical information alone. We compare the average total cost incurred by an SU in this case, to the original scenario where SUs have sensing abilities. We remark that this "no-sensing" case undermines an essential feature of cognitive radios, where the SUs are not allowed to transmit when the PU is active. Nevertheless, since spectrum sensing can be resource–consuming (especially across multiple bands), it is of theoretical interest to understand the gains provided by possessing this ability.

We start by characterizing the equilibrium behavior when the SUs do not have the ability to sense the PU's channel. Under this scenario, the SUs make a decision to use the PU band or to acquire dedicated spectrum, based only on the *unconditional average delay* experienced at the PU band. Let us first characterize this average delay under the β strategy (i.e., the strategy where each SU independently chooses to use the PU's band with probability β). First, we note that β is a Nash equilibrium strategy iff one of the following three relations hold: (i)$J_{NS}(\beta) \leq C, \ \& \ \beta = 1$; (ii)$J_{NS}(\beta) = C, \ \& \ 0 < \beta < 1$; (iii)$J_{NS}(\beta) \geq C, \ \& \ \beta = 0$. The average delay characterization is the following.

Proposition 8: The expected delay experienced by a SU at the PU band under the β strategy is given by

$$T_{NS}(\beta) = \frac{(\xi + \eta)^2 + \mu\xi}{(\xi + \eta)[\eta(\mu - \beta\lambda) - \beta\lambda\xi]}. \tag{24}$$

Proof: The proof is based on a Markov-Chain analysis where we model the PU's band as a server with breakdowns, and the arrival of a PU corresponds to a server breakdown that interrupts the service of the SU. When the PU departs, service to the SU resumes. The proof follows from the delay results in [13] on exponential servers with breakdowns. □

Once the average delay is known, the delay cost is given by $J_{NS}(\beta) = \alpha T_{NS}(\beta)$. We are now ready to ready to obtain the equilibrium point as a function of the total dedicated band cost C.

Theorem 9: The fraction of SUs that commit to the PU band at equilibrium is given as a function of the cost C by

$$\beta(C) = \begin{cases} 0 & C < J_{NS}(0) \\ \frac{\mu\eta}{\lambda(\eta+\xi)} - \frac{\alpha}{C\lambda}\left(1 + \frac{\mu\xi}{(\xi+\eta)^2}\right) & J_{NS}(0) < C < J_{NS}(1) \\ 1 & J_{NS}(1) < C \end{cases} \tag{25}$$

Proof: For $C < J_{NS}(0)$ and $C > J_{NS}(1)$, the result follows directly from the equilibrium conditions. For $J_{NS}(0) < C < J_{NS}(1)$, the expression for $\beta(C)$ follows by inverting the equation $C = \alpha T_{NS}(\beta)$ and using the expression in (24) for the delay $T_{NS}(\beta)$. □

It is easy to show that the above equilibrium point is unique for a given C.

We now compare two scenarios: either all the SUs sense the channel before making their decision (the scenario considered in Section IV) or none of them is able to do so. In view of the non-cooperative user behavior, our main objective is to examine whether the sensing capabilities improve the SUs' overall performance. It is not immediately clear if the average costs incurred by the SUs increase or decrease when sensing is possible. Indeed, in classical game-theory, it is well-known that the addition of system "resources" (radios capable of sensing in our case) sometimes increases the overall equilibrium cost. The celebrated Braess' paradox is a classic example [22], where the addition of a path might increase the congestion cost in a simple transportation network. As we prove below, a Braess-like paradox does *not* occur for the single PU band system. Nonetheless, depending on the problem parameters, having sensing capabilities can lead to the same average cost as in the case where sensing is disabled. Specifically, we show that for a certain range of dedicated spectrum cost, there is a strict advantage for the SUs in possessing the sensing ability, while for another range of spectrum cost, there is nothing to be gained (or lost) from sensing.

Let $V_S(C)$ and $V_{NS}(C)$ denote the total average cost paid by an SU, when the SUs are either equipped or not equipped with sensing. Our main result is the following.

Theorem 10: (i) If $C > J_A(1,0)$ then $V_{NS}(C) > V_S(C)$. That is, for this range of band price, there is a strict advantage for the SUs in being able to sense the presence of a PU.

(ii) If $C < J_A(1,0)$ then $V_{NS}(C) = V_S(C)$. That is, sensing ability does not lead to a cost advantage, but neither to a disadvantage.

Proof: The proof follows by first obtaining explicit expressions for $V_S(C)$ (Lemma 5 below) and $V_{NS}(C)$ (Lemma 6 below), which are straightforward to show.

Lemma 5: If the SUs posses the ability to sense the presence of a PU, the average cost per unit time paid by the SUs is given by

$$V_S(C) = \begin{cases} \lambda\frac{\xi C+\eta J_A(1,q(C))}{\eta+\xi} & J_O(1,0) < C < J_O(1,1) \\ \lambda\frac{\xi C+\eta J_A(1,0)}{\eta+\xi} & J_A(1,0) < C < J_O(1,0) \\ \lambda C & J_A(0,0) < C < J_A(1,0) \end{cases}$$

Proof: Follows along the same lines as Proposition 6, using the equilibrium characterization in Theorem 6. □

Lemma 6: If the SUs lack the ability to sense the presence of a PU, the average cost per unit time paid by the SUs is given by

$$V_{NS}(C) = \begin{cases} \lambda J_{NS}(1) & J_{NS}(1) < C < J_O(1,1) \\ \lambda C & J_A(0,0) < C < J_{NS}(1) \end{cases}$$

Proof: If $J_{NS}(1) < C < J_O(1,1)$, then all the SUs take the PU band, and each of them pays a delay cost $J_{NS}(1)$. On the other hand, if $J_A(0,0) < C < J_{NS}(1)$, the SUs are either divided between two equally costly options, or they all take the dedicated spectrum. It easily follows that the cost paid in either case is C per user. □

We are now ready to prove the theorem. Part (ii) follows immediately from Lemmas 5 and 6. Let us prove part (i) by dividing up the cost range. First consider $J_{NS}(1) < C < J_O(1,1)$. Using Lemmas 5 and 6, we have

$$V_{NS}(C) = J_{NS}(1) = \frac{\eta J_A(1,1) + \xi J_O(1,1)}{\eta + \xi} > \frac{\eta J_A(1,q(C)) + \xi J_O(1,q(C))}{\eta + \xi} = V_S(C),$$

where the last equality follows from $J_O(1,q(C)) = C$. Second, for the range $J_O(1,0) < C < J_{NS}(1)$, we have $V_{NS}(C) = \lambda C = \frac{\eta\lambda C + \xi\lambda C}{\eta+\xi} > \frac{\eta\lambda J_A(1,q(C)) + \xi\lambda C}{\eta+\xi} = V_S(C)$. Finally, for the range $J_A(1,0) < C < J_O(1,1)$, $V_{NS}(C) = \lambda C = \frac{\eta\lambda C + \xi\lambda C}{\eta+\xi} > \frac{\eta\lambda J_A(1,0) + \xi\lambda C}{\eta+\xi} = V_S(C)$. □

Note that according to the theorem, there is never a strict cost disadvantage in possessing the ability to sense, and hence there is no Braess'-like paradox. Yet, while there is a strict cost improvement for the range $C > J_A(1,0)$, there is no improvement for the range $C < J_A(1,0)$.

VII. Concluding Remarks

In this paper, we considered the decision-making process of Secondary Users who have the option of either acquiring dedicated spectrum or sharing free yet unreliable bands. We fully characterized the resulting Nash equilibrium for the single-band case. We also demonstrated how the equilibrium analysis can be exploited from the viewpoint of a monopoly who owns dedicated spectrum and wishes to maximize revenue. Furthermore, we examined the effect of the spectrum sensing ability on the resulting equilibrium.

Overall, this paper uses a novel paradigm to provide a first step towards a theoretical understanding of decision processes in dynamic spectrum access systems. Our study integrates tools and ideas from queuing theory, game theory, and network economics. There are still many problems and extensions that can be dealt with. For example, we plan to extend the model to account for other distributions beside the exponential distribution. Moreover, in future work, we plan to incorporate into the model additional costs associated with using free spectrum, e.g., the energy-consumption cost related to spectrum sensing. Overhead costs associated with renting dedicated spectrum can be considered as well, such as the cost of communication during the rent agreement, and congestion effects when dedicated spectrum is not widely available. For multiple PU bands, one may consider SUs with *partial* sensing abilities (e.g., may sense only a subset of the bands) and their effect on the performance. It is also of interest to analyze scenarios in which the dedicated spectrum is owned by multiple providers that compete over the spectrum market share (e.g., the model of [34]).

In this paper, we have considered basic decision-making of SUs, who choose between dedicated or free spectrum upon arrival. It is also of interest to examine more sophisticated decision sets and user types, for example, impatient SUs who purchase a dedicated band whenever their waiting time for free spectrum exceeds some threshold. This would naturally require extending the user model, perhaps by building on call-center research (see, e.g., [42]). Finally, as indicated in the IEEE 802.22 standard, white spaces can be allocated either by employing MAC protocols or through a spectrum broker, which divides the available bandwidth between the SUs. Studying the former model requires encapsulating the analysis of distributed MAC protocols within our framework. For the latter model, we plan to consider the case in which the broker allocates the spectrum band to the SUs and also announces the congestion levels for potential SUs.

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APPENDIX

On the correctness of the ASTA property. We argue that conditioned upon a PU being absent (or present), an arriving SU sees time average occupancies, i.e., that it sees the same conditional distribution as an external observer. In other words, we will show that $\overline{N}_A = \hat{N}_A$ and $\overline{N}_O = \hat{N}_O$. We argue along the same lines as in [43]. Let us first condition on the PU being present. For some small $\delta > 0$, let $A(t, t+\delta)$ denote the event that a SU arrives in the time interval $(t, t+\delta)$. As shown in [43], the ASTA property would hold conditioned on the PU being present if the following condition is satisfied.

$$\mathbb{P}\{A(t,t+\delta)|N_O(t)=n\} = \mathbb{P}\{A(t,t+\delta)\}. \tag{26}$$

Now, conditioned on the PU being present, the arrival process is Poisson with rate $q\lambda$, and therefore arrival event $A(t,t+\delta)$ is independent of how many packets there are in the system. Thus, (26) holds under in this case. A similar argument would prove that ASTA also holds conditioned on the PU being absent.

In order to illuminate the situation further, we show that ASTA does not hold for an *unconditional* arrival. Consider an arbitrary arrival into the queue. It is not known whether the PU is present or not. Suppose for the sake of easy argument that $p\lambda$ is very small, $q\lambda$, and μ are very large, and that $\eta = \xi$ and both are very small compared to μ. In such a case, we can deduce from (11) and (12) that large queue occupancies are likely when the PU is present and small occupancies are typical when the PU is absent. Therefore, the conditional probability $\mathbb{P}\{A(t,t+\delta)|N(t)=n\}$ is actually dependent on n. For example, conditioned on a very large occupancy, it is more likely that the PU is present, so that the probability of an arrival is closer to $q\lambda\delta$, whereas, the unconditional probability of arrival is given by

$$\mathbb{P}\{A(t,t+\delta)\} = \delta\frac{p\lambda\eta + q\lambda\xi}{\eta+\xi}.$$

Thus, $\mathbb{P}\{A(t,t+\delta)|N(t)=n\} \neq \mathbb{P}\{A(t,t+\delta)\}$, and ASTA does not hold for an unconditional arrival.