

# Detection of False Data Injection Attacks in Power Systems Using a Secured-Sensors and Graph-Based Method

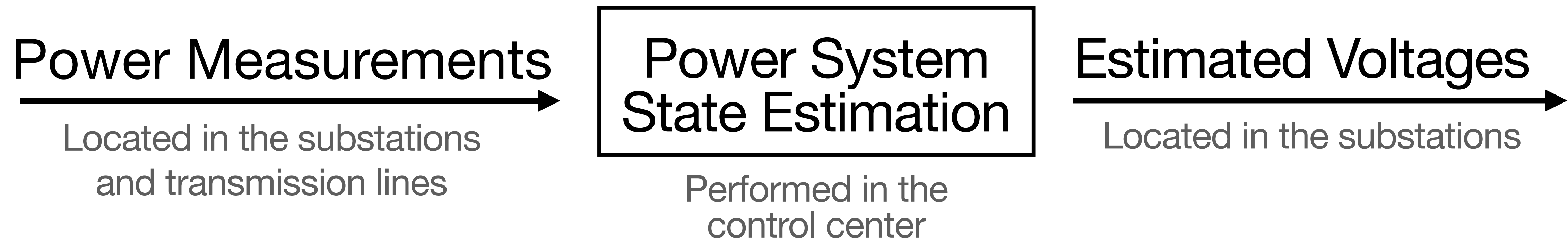
**G. Morgenstern<sup>1</sup>, L. Dabush<sup>1</sup>, J. Kim<sup>2</sup>, J. Anderson<sup>3</sup>, G. Zussman<sup>3</sup>, T. Rottenburg<sup>1</sup>**

<sup>1</sup> Ben-Gurion University of the Negev, Beer-Sheva, Israel

<sup>2</sup> Kentech, Naju-si, South Korea

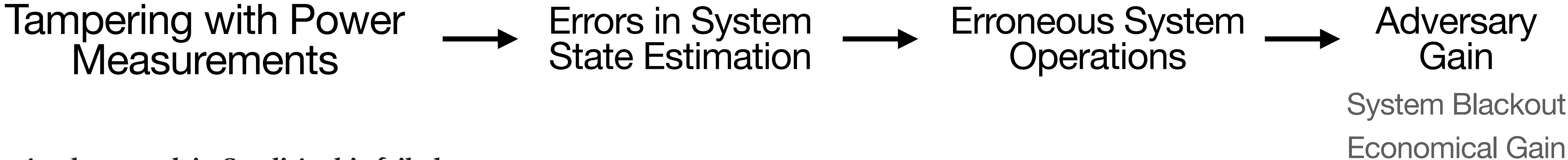
<sup>3</sup> Columbia University, New York, USA

# Power System State Estimation



Transmitting measurements via cyber communication is prone to cyber attacks

# False Data Injection Attacks



## A cyber attack in Saudi Arabia failed to cause carnage, but the next attempt could be deadly

At a time when the world faces a dangerous escalation in cyber warfare, a series of assaults on petrochemical companies in Saudi Arabia – possibly backed by nation states – has caused alarm

Nicole Perloth, Clifford Krauss • Tuesday 20 March 2018 14:14 • [Comments](#)



Computers crashed at sites including Sadara Chemical Company, a joint venture between the oil and chemical giants Saudi Aramco and Dow Chemical (Sadara)

TECHNOLOGY NEWS JANUARY 18, 2017 / 1:06 PM / UPDATED 7 YEARS AGO

## Ukraine's power outage was a cyber attack: Ukrenergo

By Pavel Polityuk, Oleg Vukmanovic, Stephen Jewkes

3 MIN READ  

KIEV/MILAN (Reuters) - A power blackout in Ukraine's capital Kiev last month was caused by a cyber attack and investigators are trying to trace other potentially infected computers and establish the source of the breach, utility Ukrenergo told Reuters on Wednesday.

# Defense Against False Data Injections

## Option 1: Attack prevention

Requires additional resources

## Option 2: Attack detection

Relies on intrinsic system and attack properties

# Outline

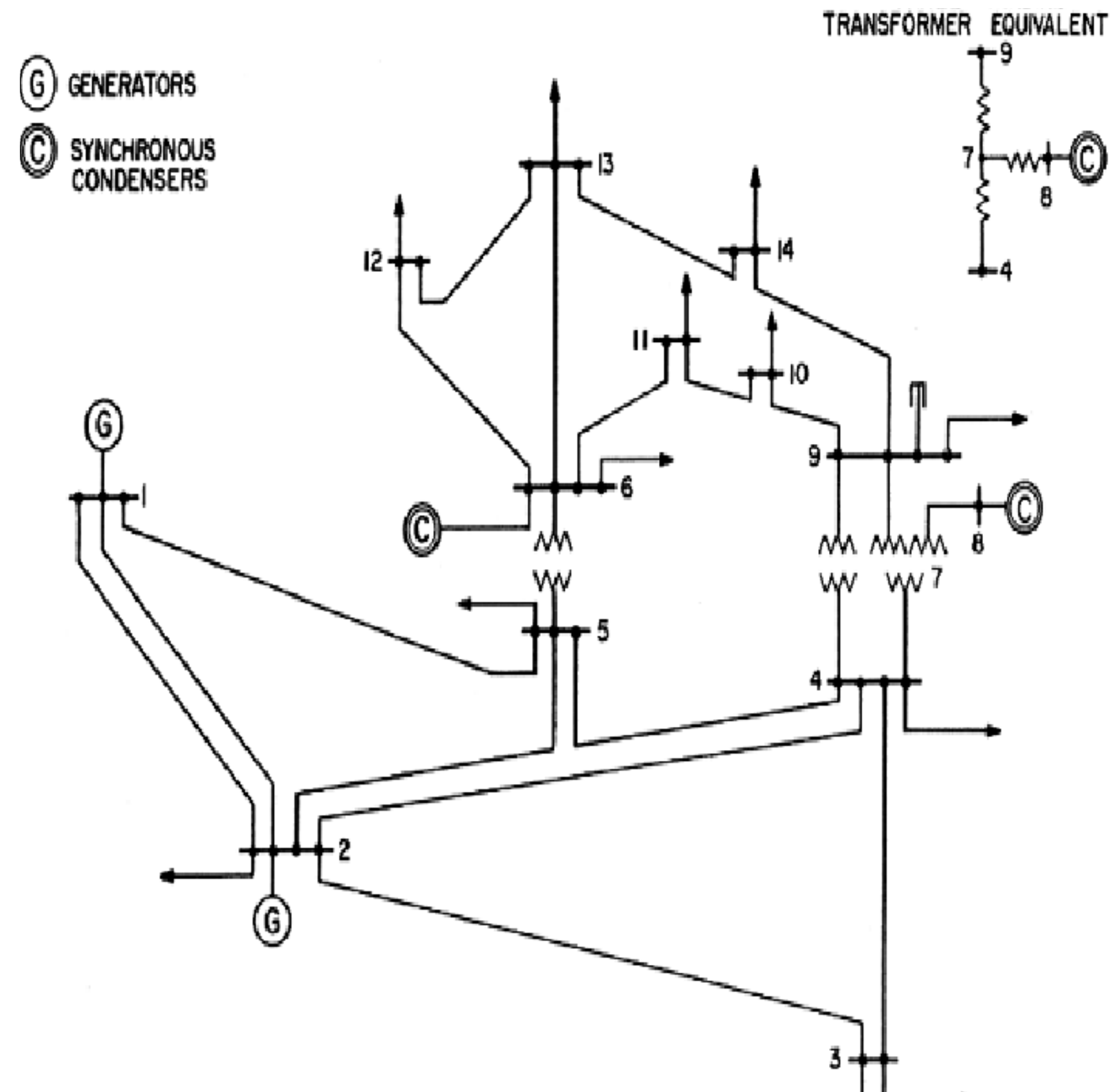
- Introduction
- **Background**
- Theory: Secured Sensors and Graph Smoothness Based Detection for False Data Injection Attacks
- Theory: Modification for Distributed Optimization
- Theory: Modification to Graph Low Pass Signals
- Performance Evaluation

# Power System Represented as an Undirected Graph

Substations (generators/loads)

Transmission lines

Susceptance over the lines

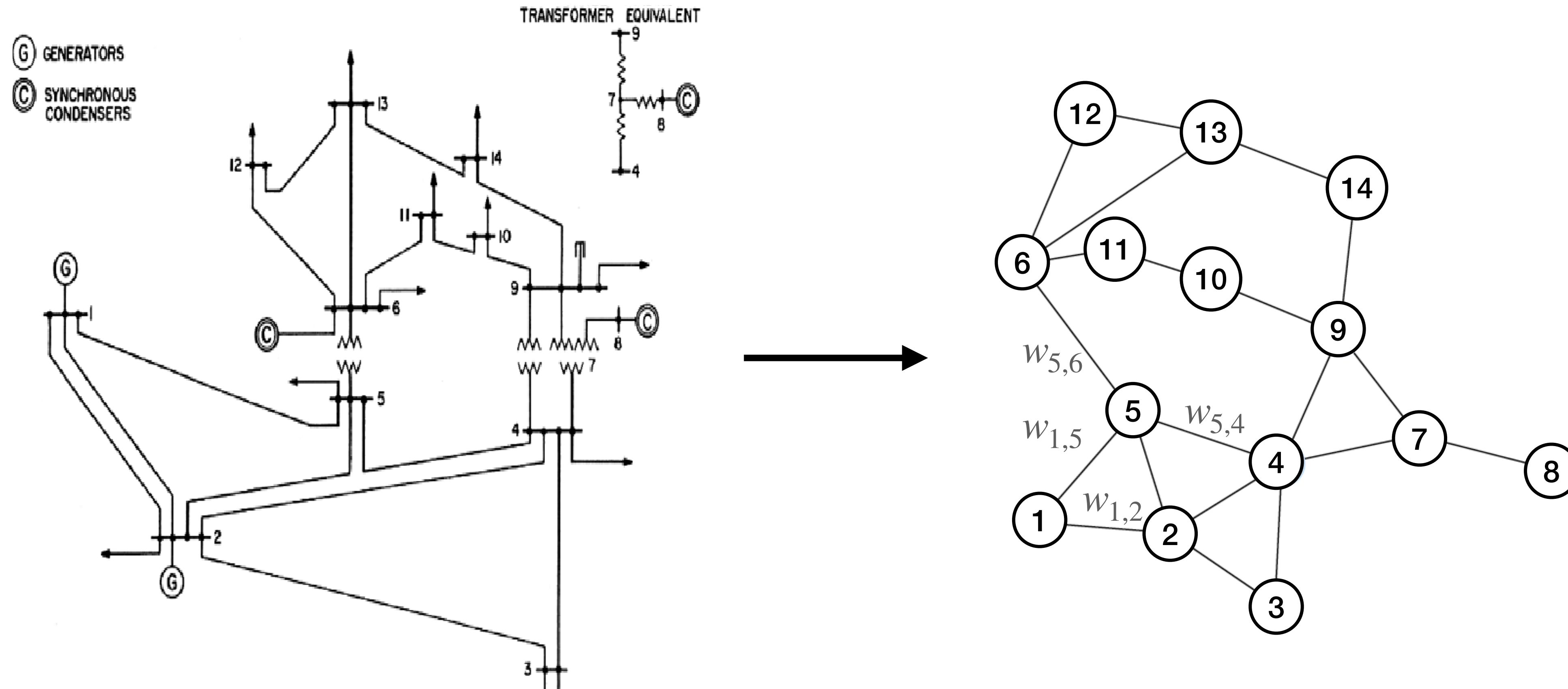


# Power System Represented as an Undirected Graph

Substations (generators/loads)  $\rightarrow$  vertices

Transmission lines  $\rightarrow$  edges

Susceptance over the lines  $\rightarrow$  edge weights



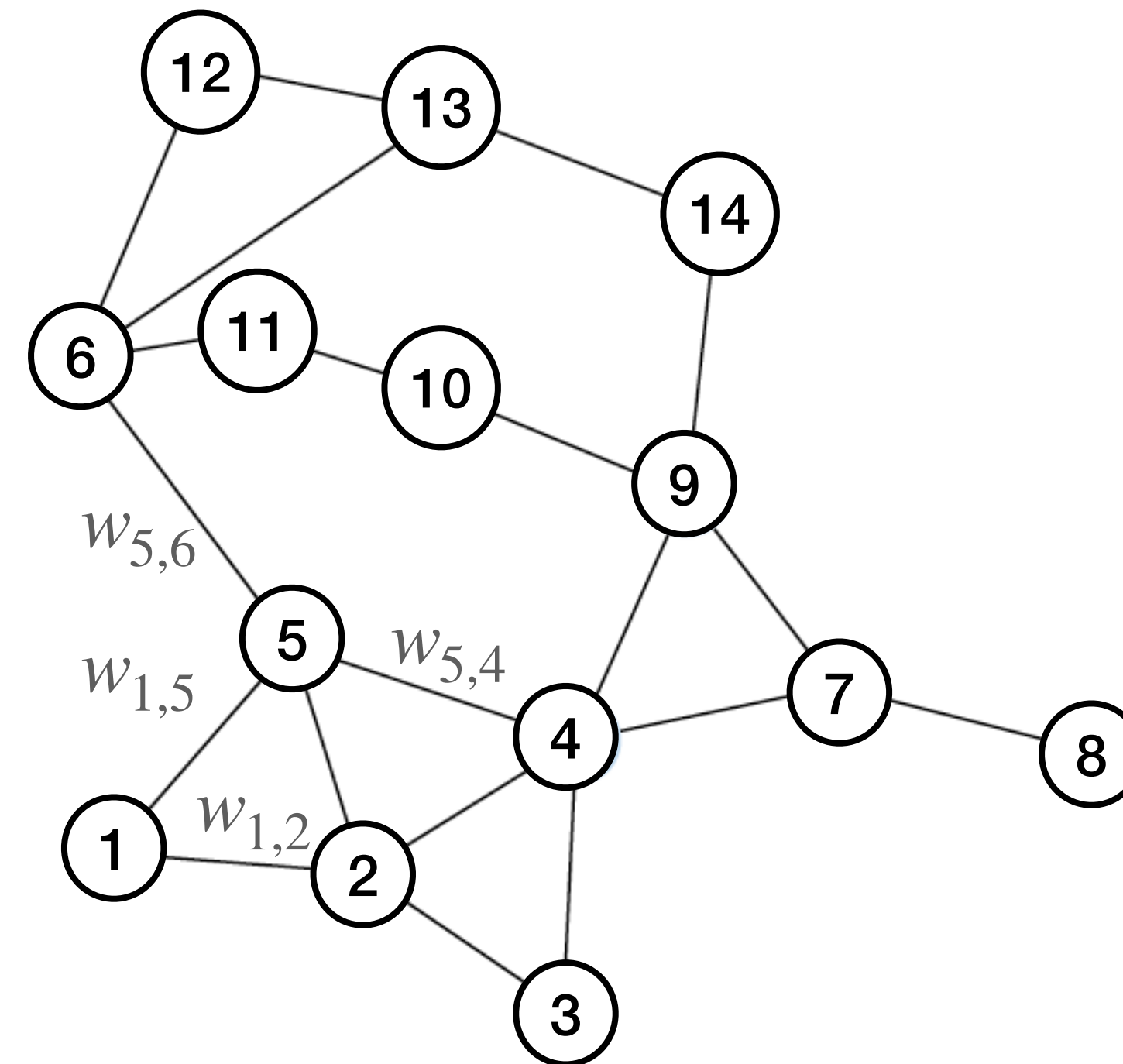
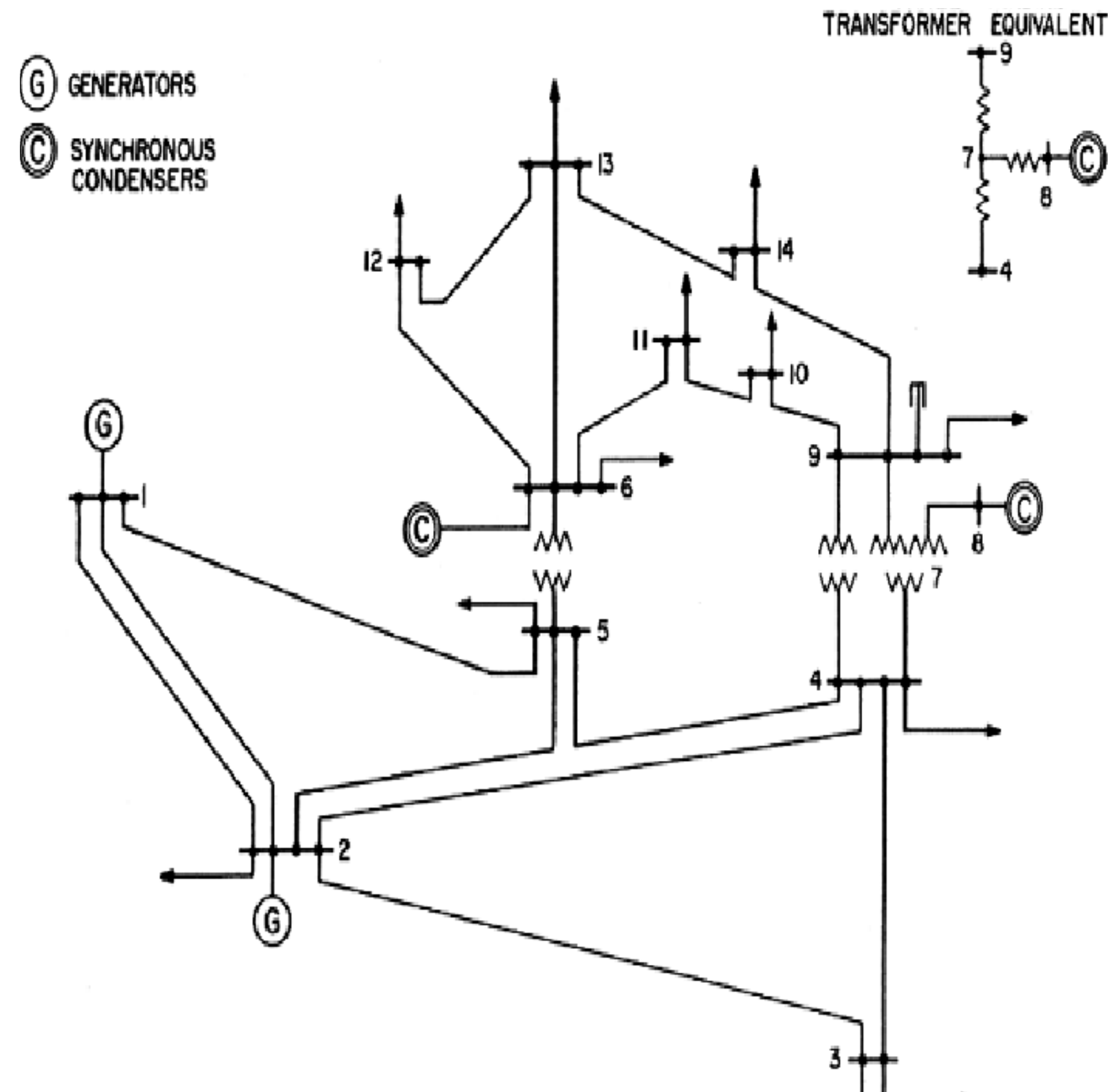
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Susceptance over the lines  $\rightarrow$  edge weights

(generators/loads)  $\longleftrightarrow$  (Sources/Sinks)





# Direct Current Power Flow Model

Vertex measurement: (active power injection)

$$z_v = \sum_{u \in \mathcal{N}_v} w_{v,u} (\theta_v - \theta_u)$$

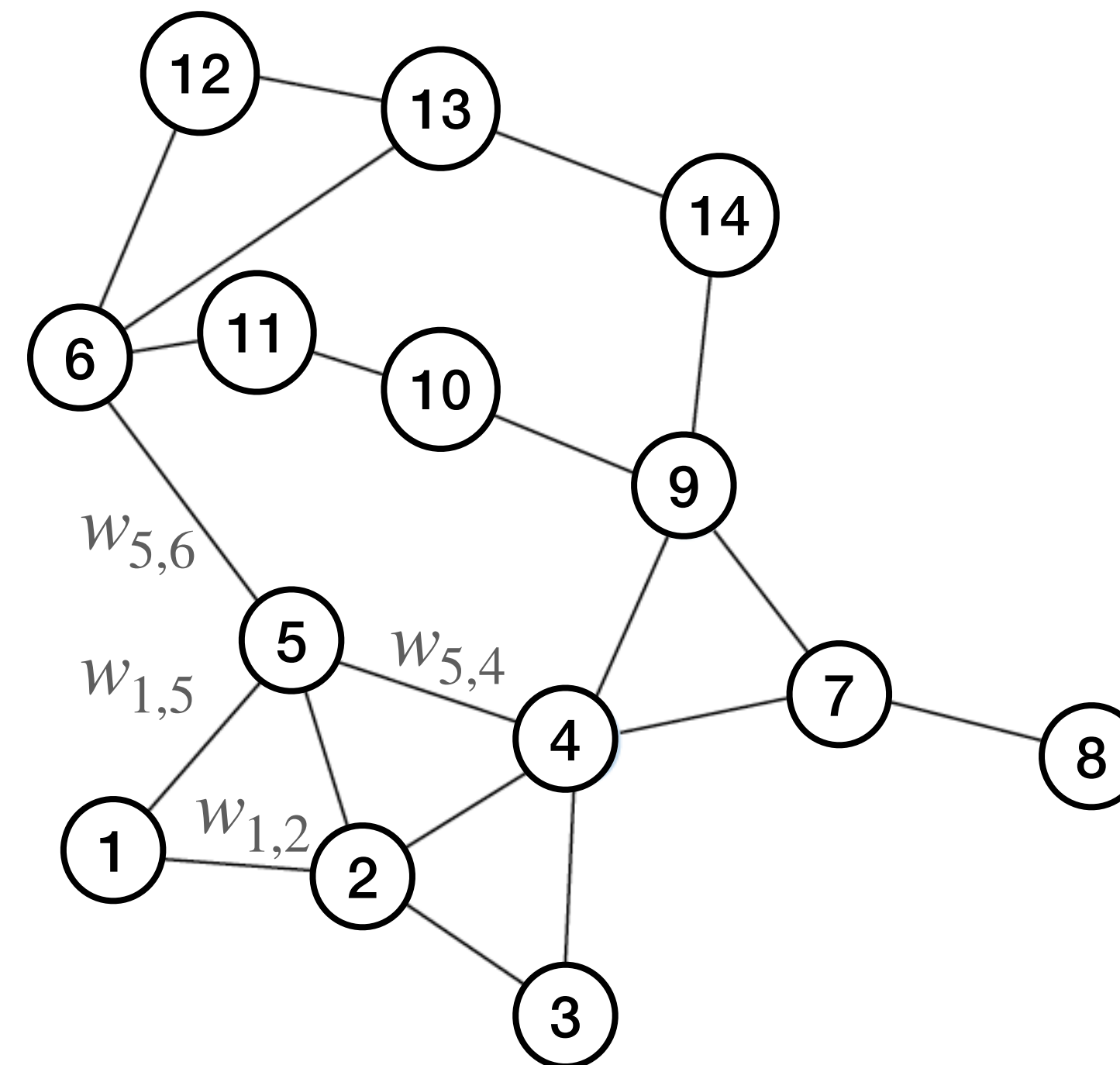
Edge measurement: (active power flow)

$$z_{(u,v)} = w_{v,u} (\theta_v - \theta_u)$$

$\mathcal{N}_v$ : neighbor vertices of vertex  $v$

$w_{u,v}$ : weight over edge  $(u, v)$

$\theta_v$ : state value over vertex  $v$  (voltage phase)



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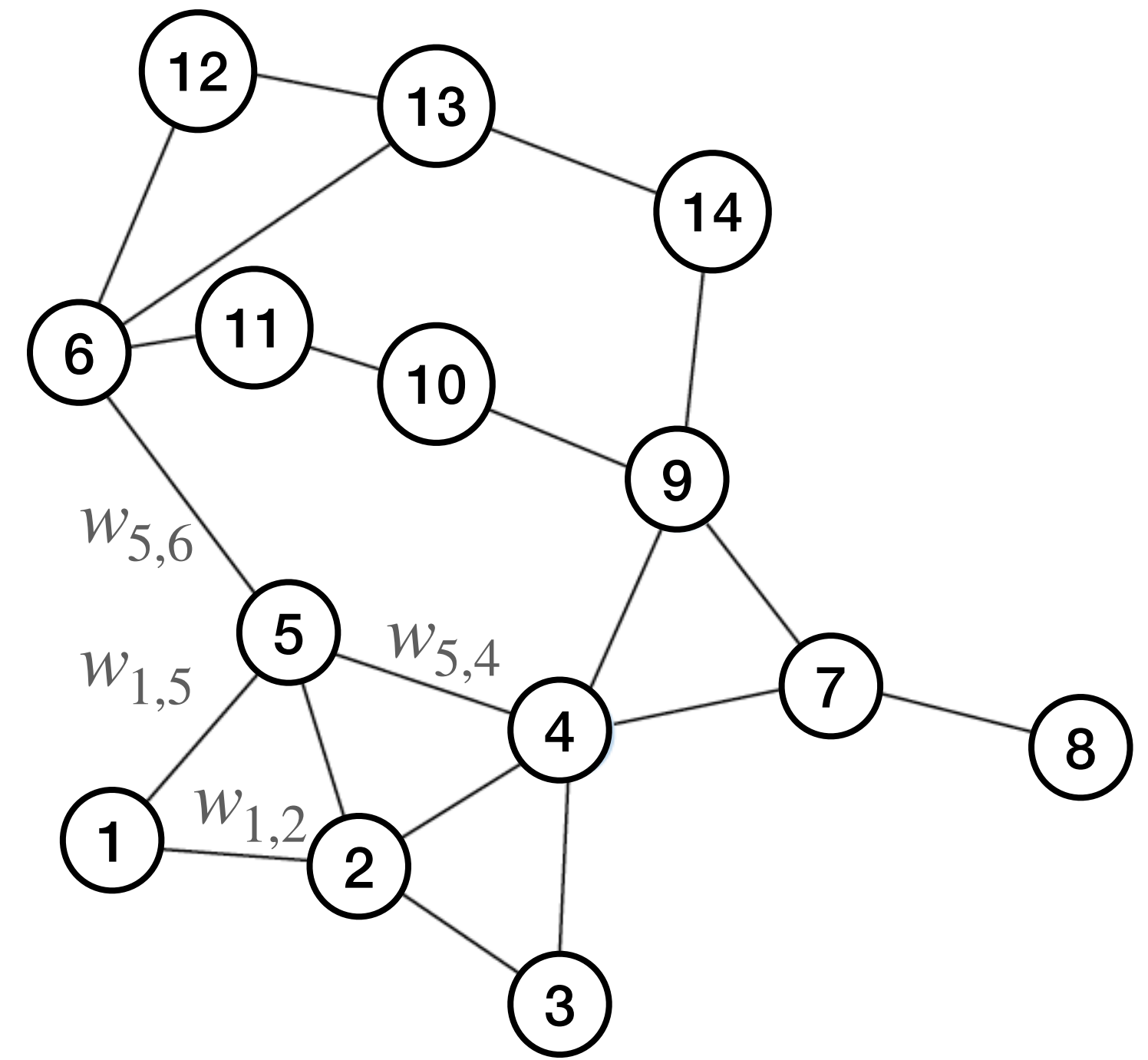
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Linear Model

$$z = H\theta + noise$$

$H$  - represents the system topology

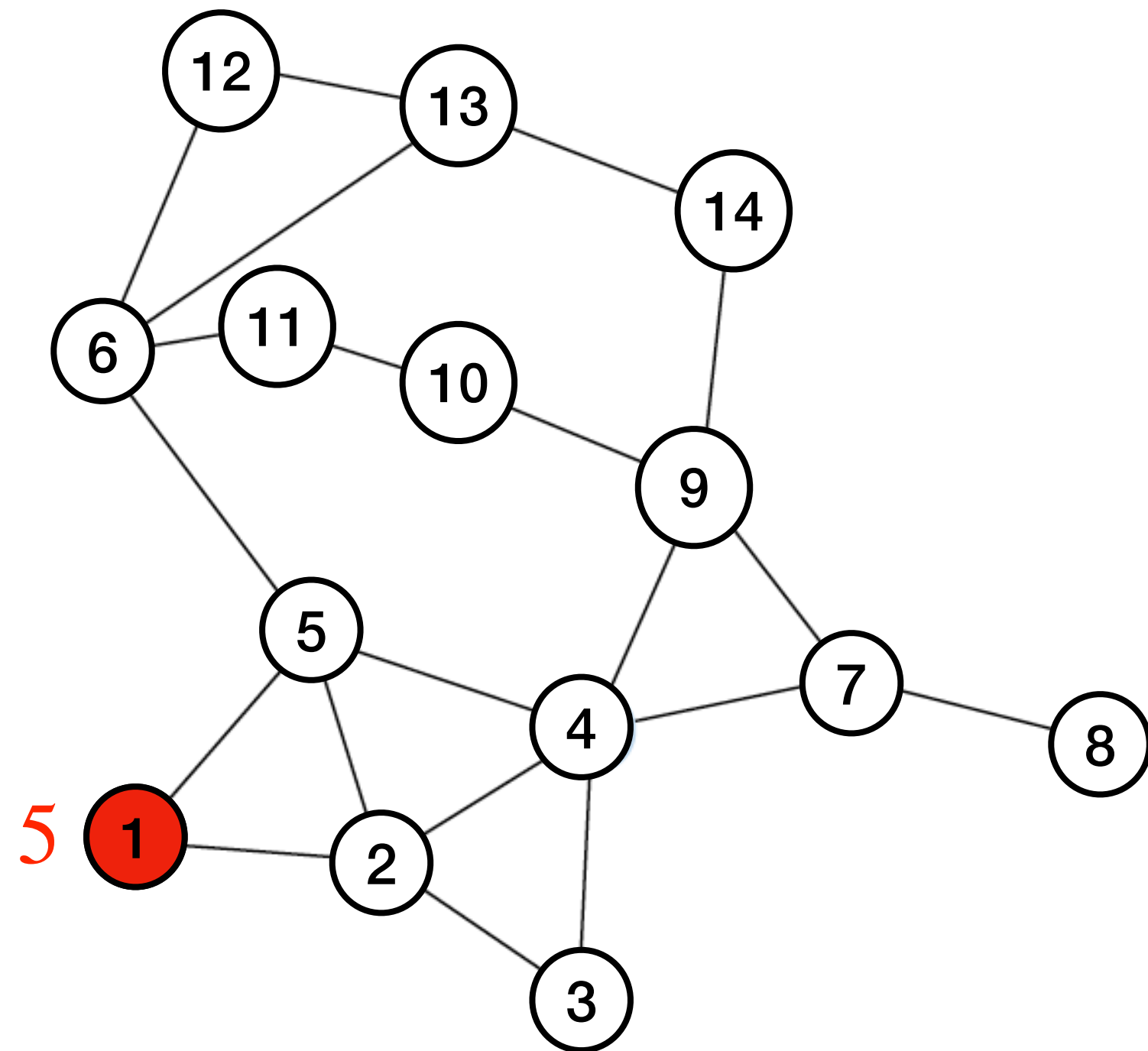
# False Data Injection Attack Models

$$z = H\theta + a + noise$$

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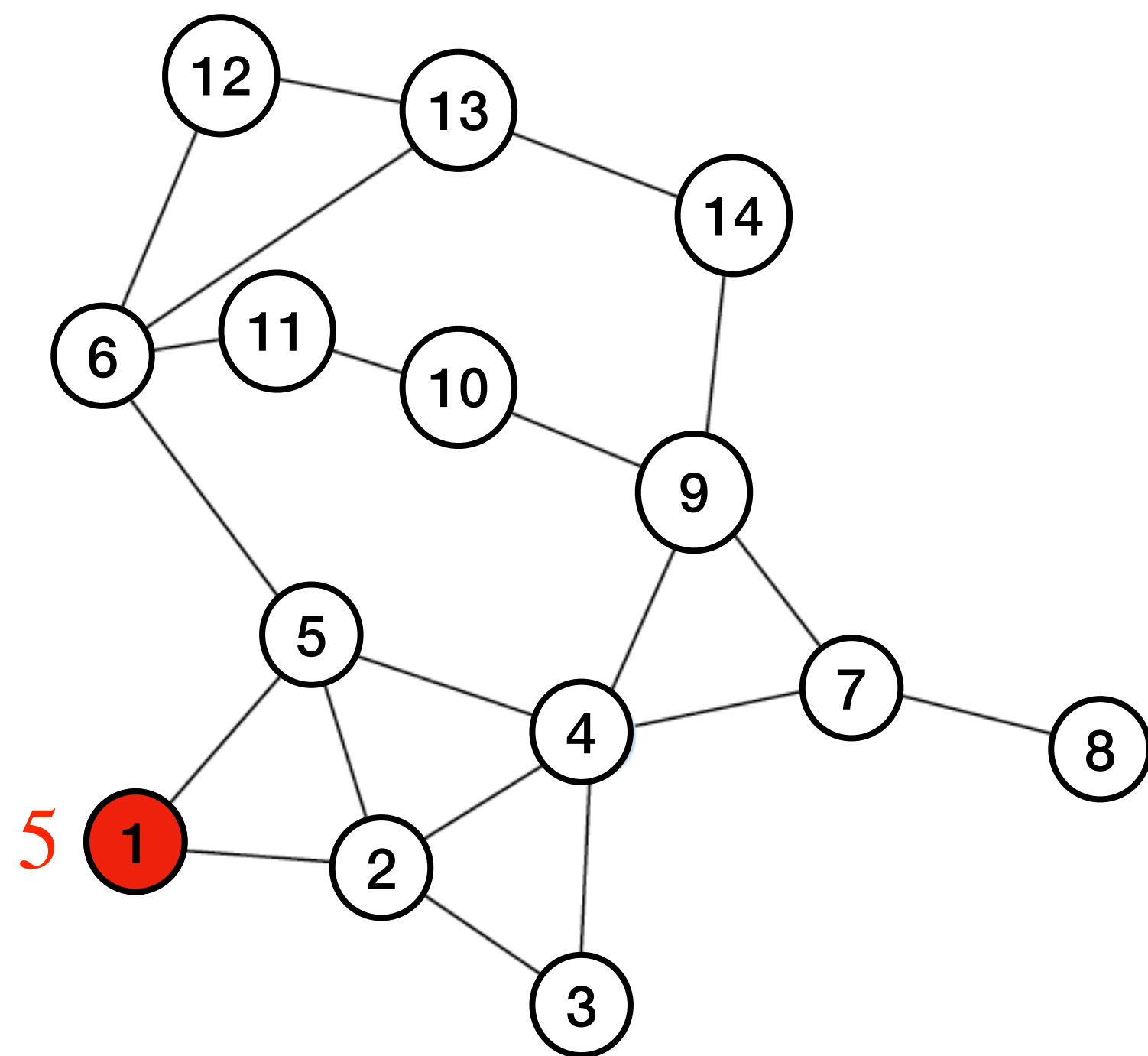
Regular: *a*



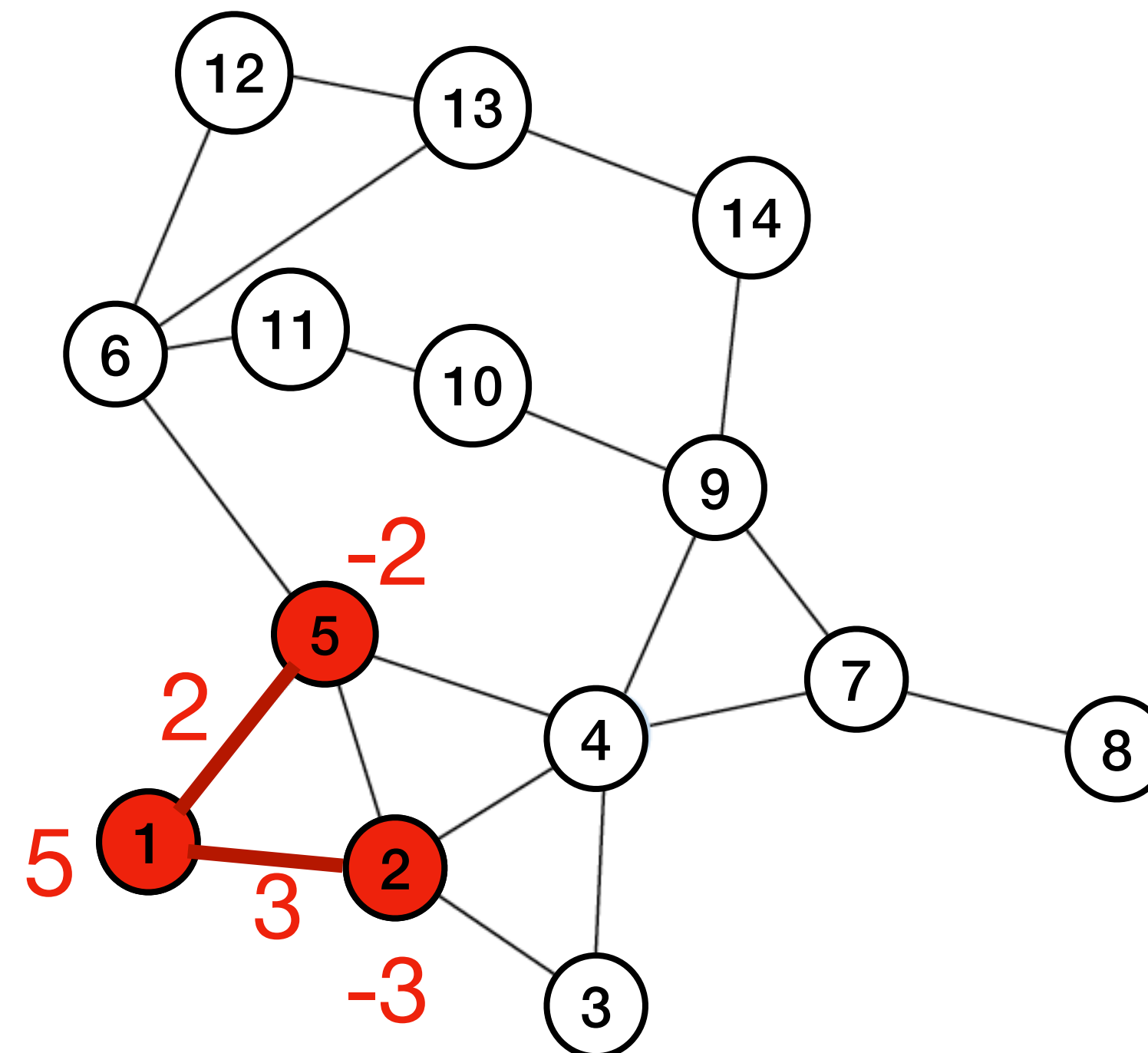
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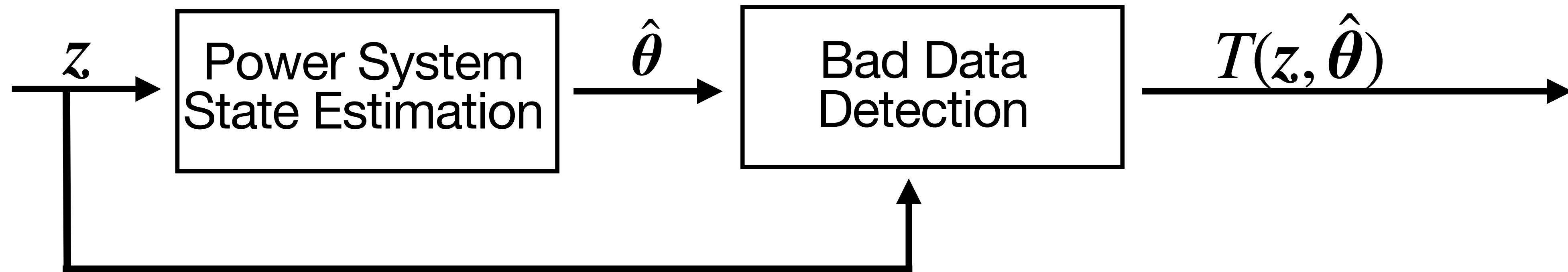


Unobservable:  $a = Hc$



# Traditional Bad Data Detection

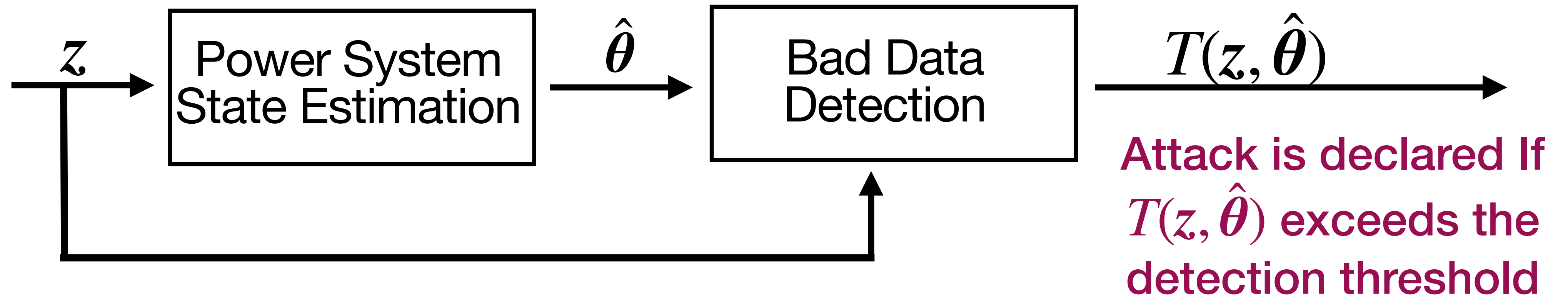
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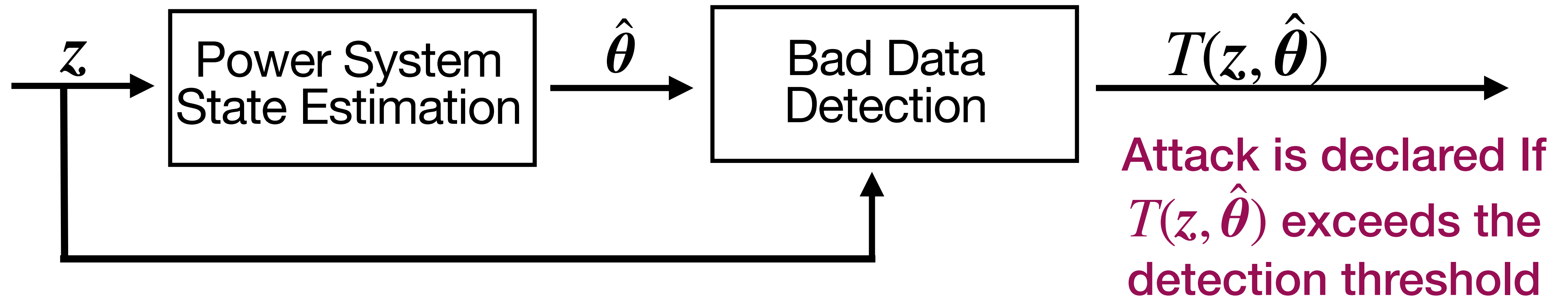
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# Traditional Bad Data Detection

$$z = H\theta + a + noise$$



$$T(z, \theta) = \|z - H\theta\|_2^2$$

$$\hat{\theta} = \min_{\theta} T(z, \theta) = (H^T H)^{-1} H^T z$$

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# Traditional Bad Data Detection

Fail to detect unobservable false data injection attacks ( $a = Hc$ )

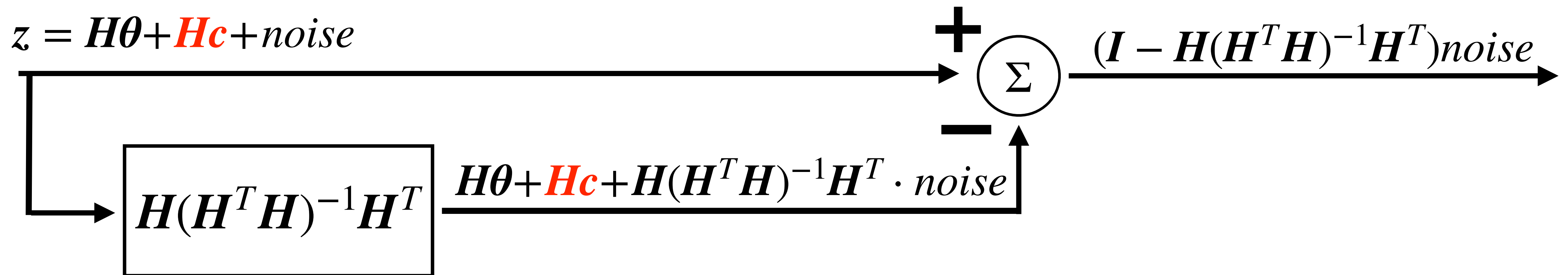
$$\begin{aligned} T(\mathbf{z}, \hat{\boldsymbol{\theta}}) &= \|\mathbf{z} - \mathbf{H}\hat{\boldsymbol{\theta}}\|_2^2 \\ &= \|\mathbf{z} - \mathbf{H}(\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T \mathbf{z}\|_2^2 \end{aligned}$$

Liu, Y., Ning, P., & Reiter, M. K. (2011). False data injection attacks against state estimation in electric power grids. *ACM Transactions on Information and System Security (TISSEC)*, 14(1), 1-33.

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# Outline

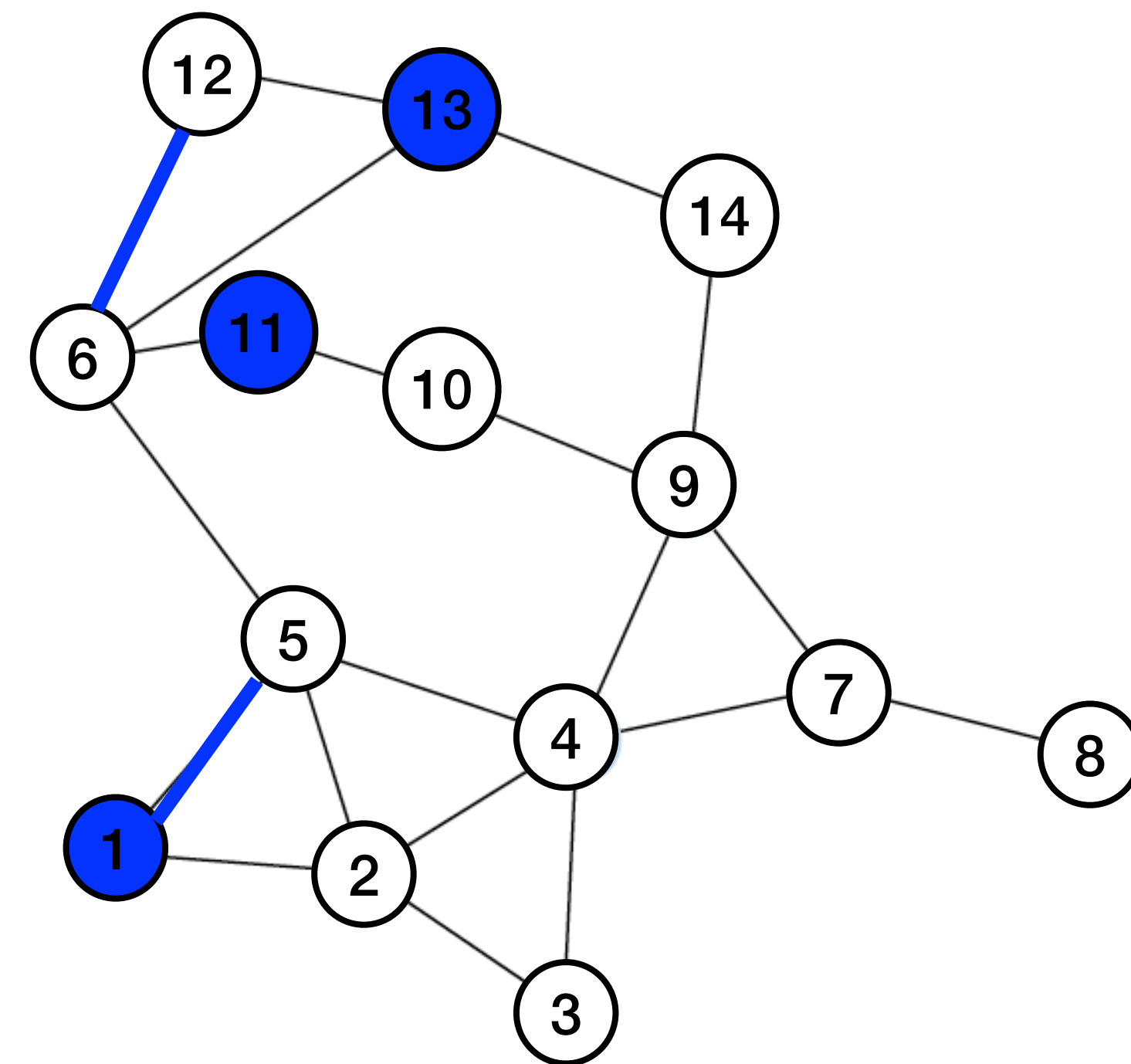
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# Secured Sensors

$$a_{\mathcal{S}} = \mathbf{0}$$

Secured sensors are assumed to be immuned from an attack

Additional resources are used to protect these sensors: guards, electric fences, .....



$$\mathcal{S} = \{1, 11, 13, (1, 5), (6, 12)\}$$

# Secured Sensors

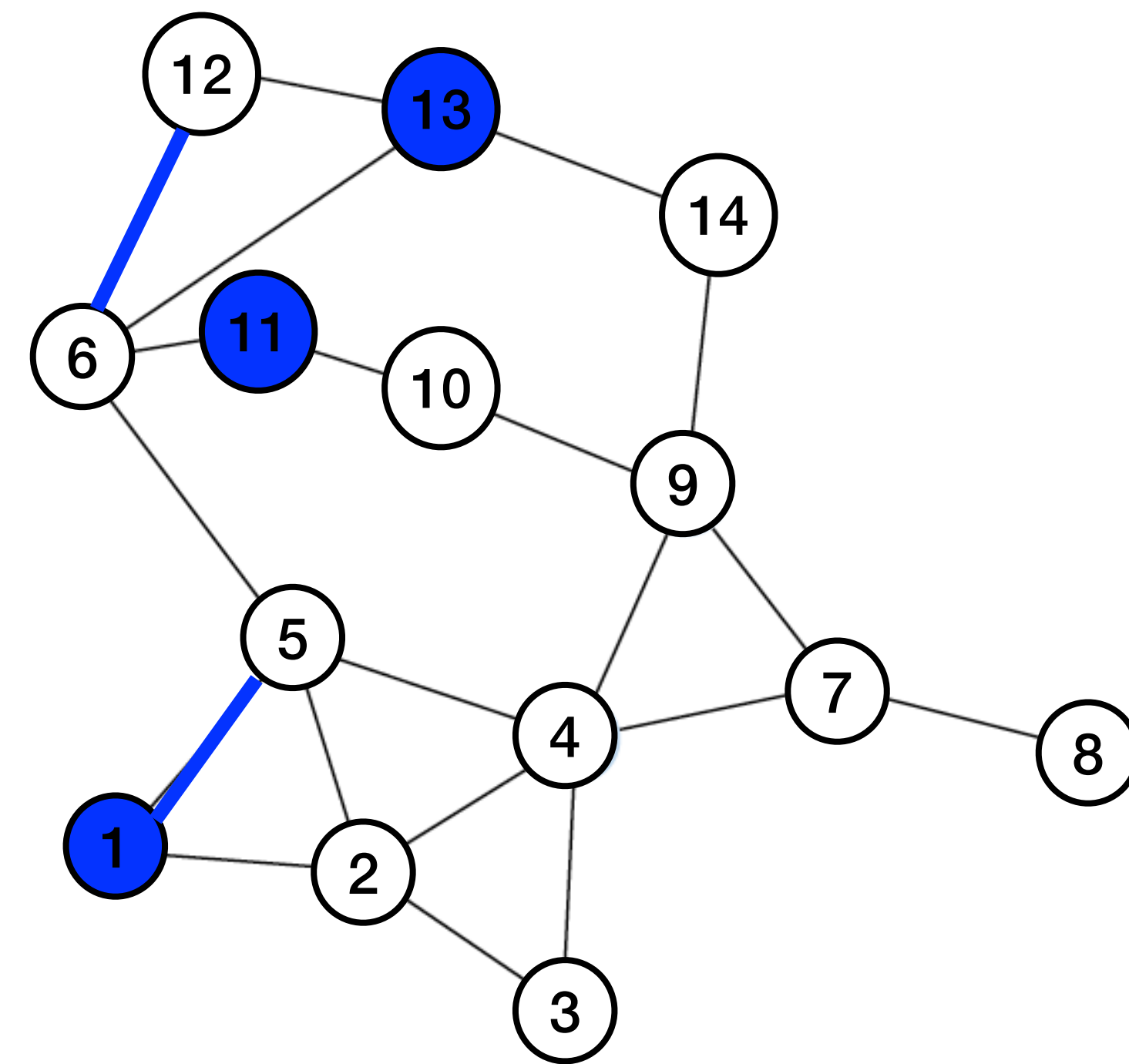
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Design Flexibility

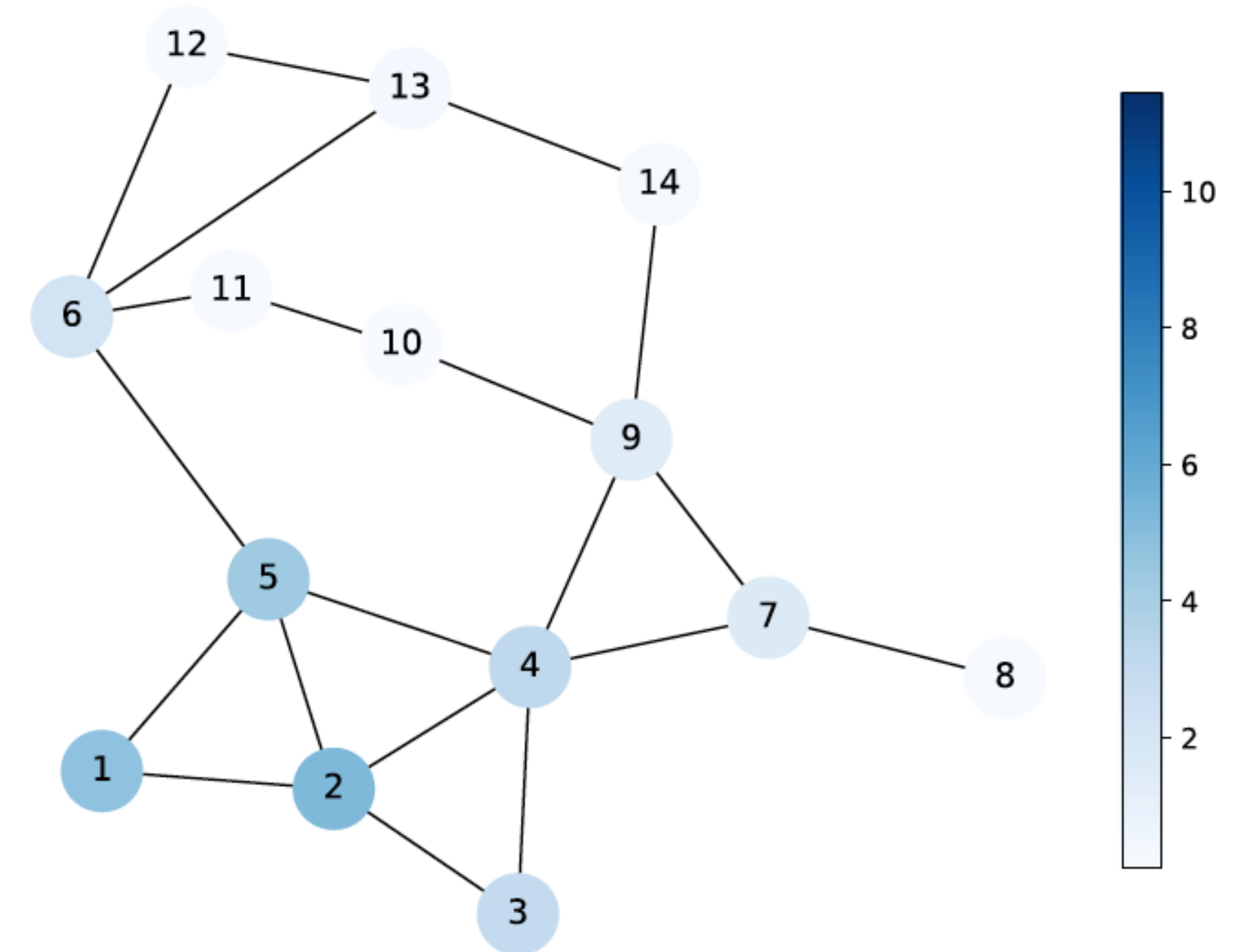
$$a_{\mathcal{S}} = \mathbf{0} \rightarrow \|a_{\mathcal{S}}\|_2^2 \leq \epsilon_1$$



$$\mathcal{S} = \{1, 11, 13, (1, 5), (6, 12)\}$$

# Power System States are Smooth Graph Signals

The difference between the signal state values in neighbor vertices is assumed small



## State Signal

Each vertex is assigned with a value represented by its color

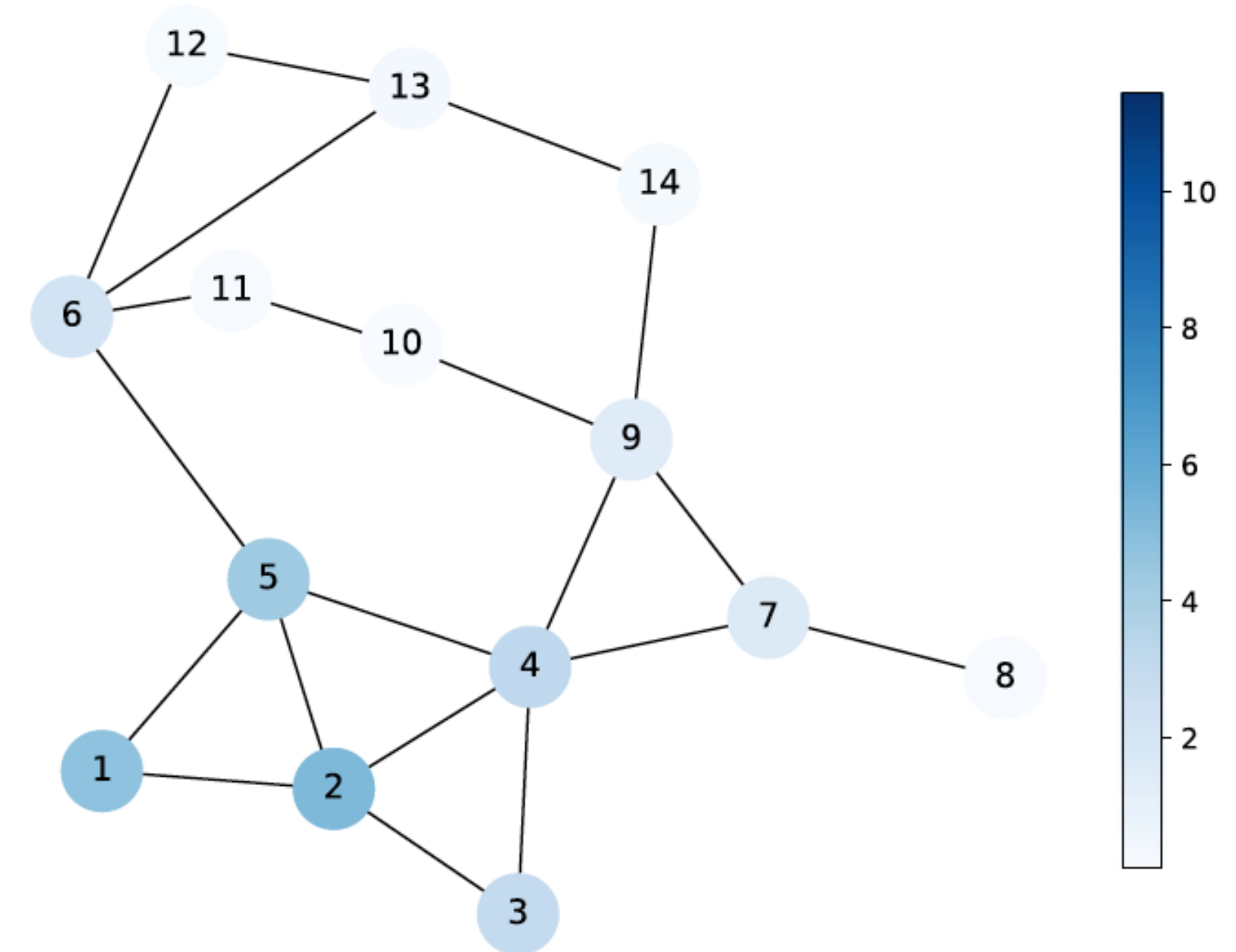
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It has a bounded graph Total Variation

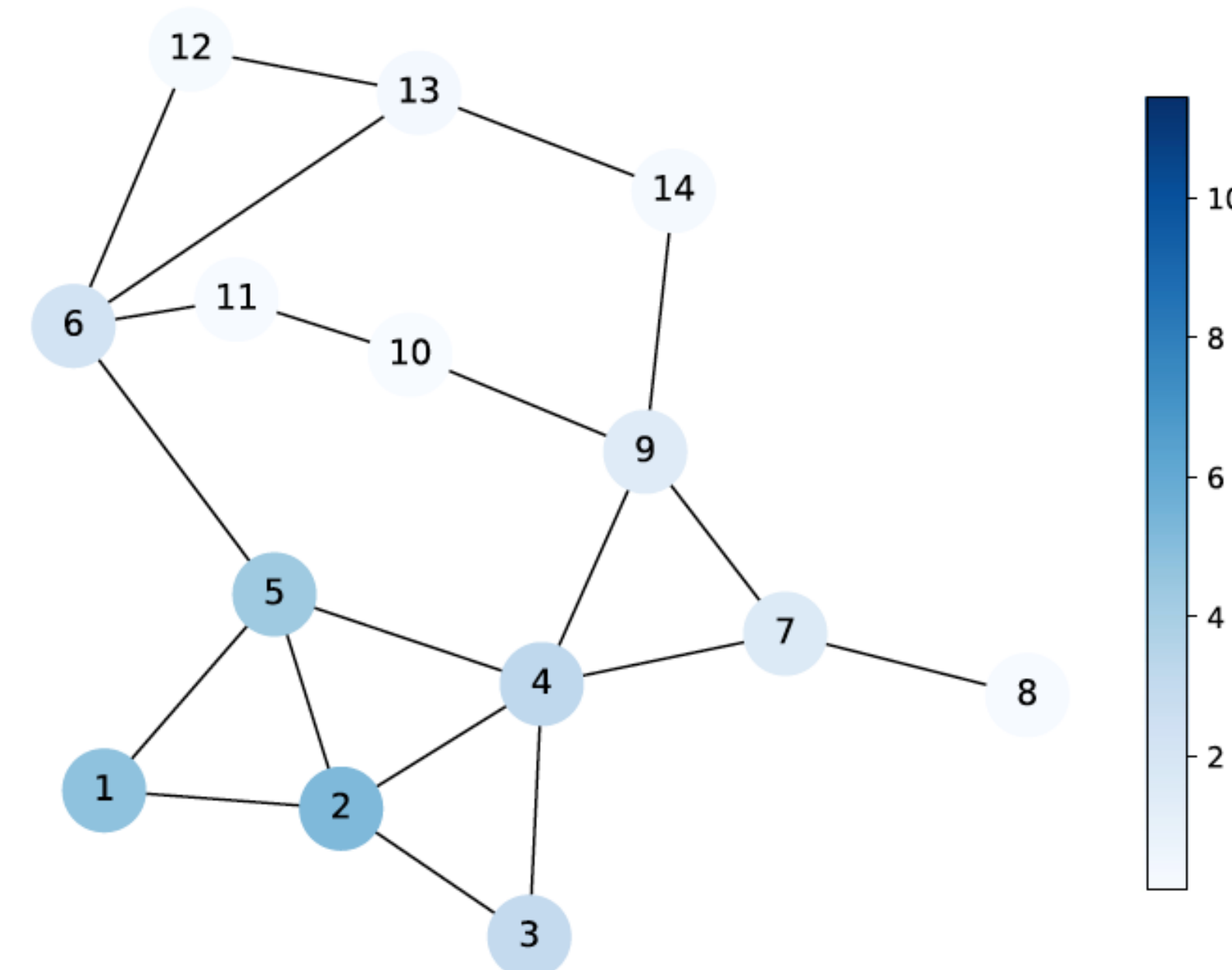
$$\sum_{v \in \mathcal{V}} \sum_{u \in \mathcal{N}_v} w_{v,u} (\theta_u - \theta_v)^2 \leq \epsilon_2$$

Sum over system vertices

Sum over vertex  $v$  neighbors

Edge weight

Signal variation between vertex  $v$  and vertex  $u$



## State Signal

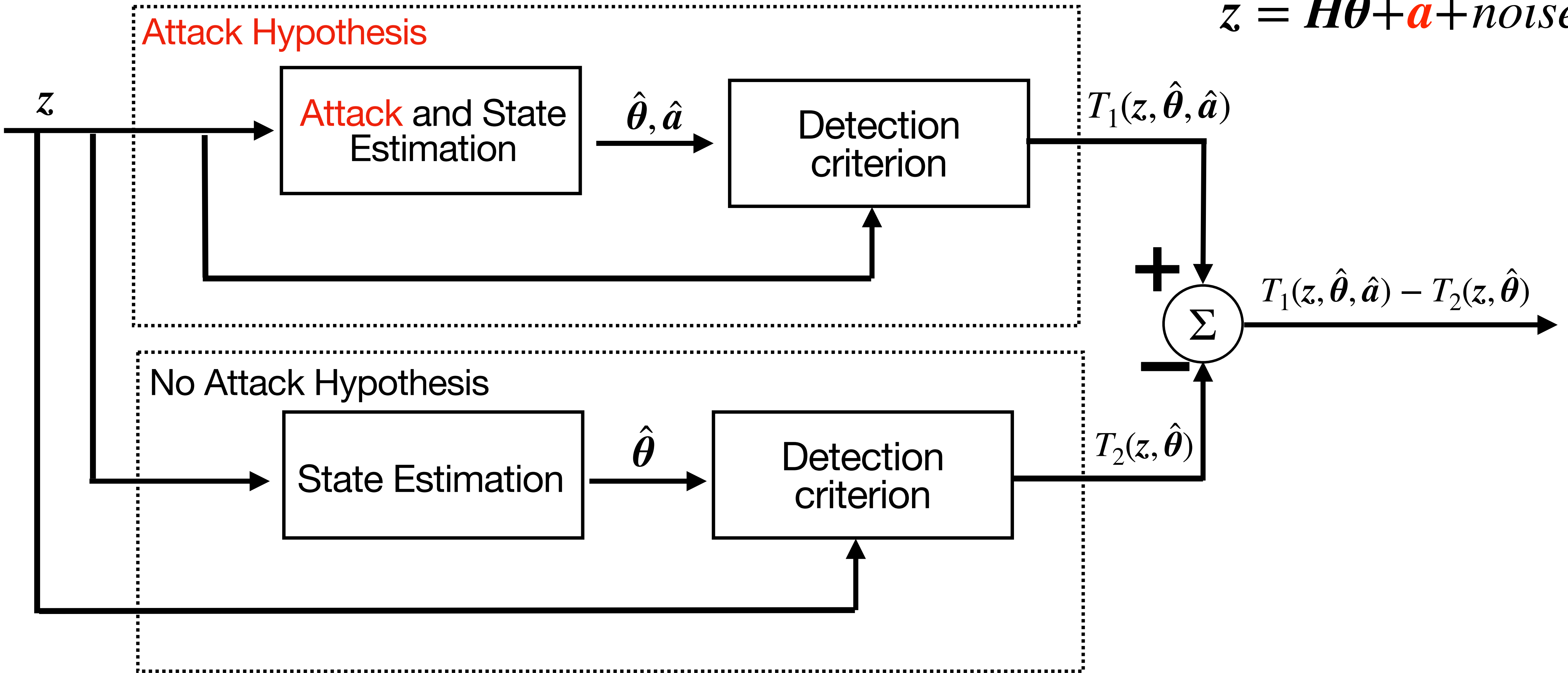
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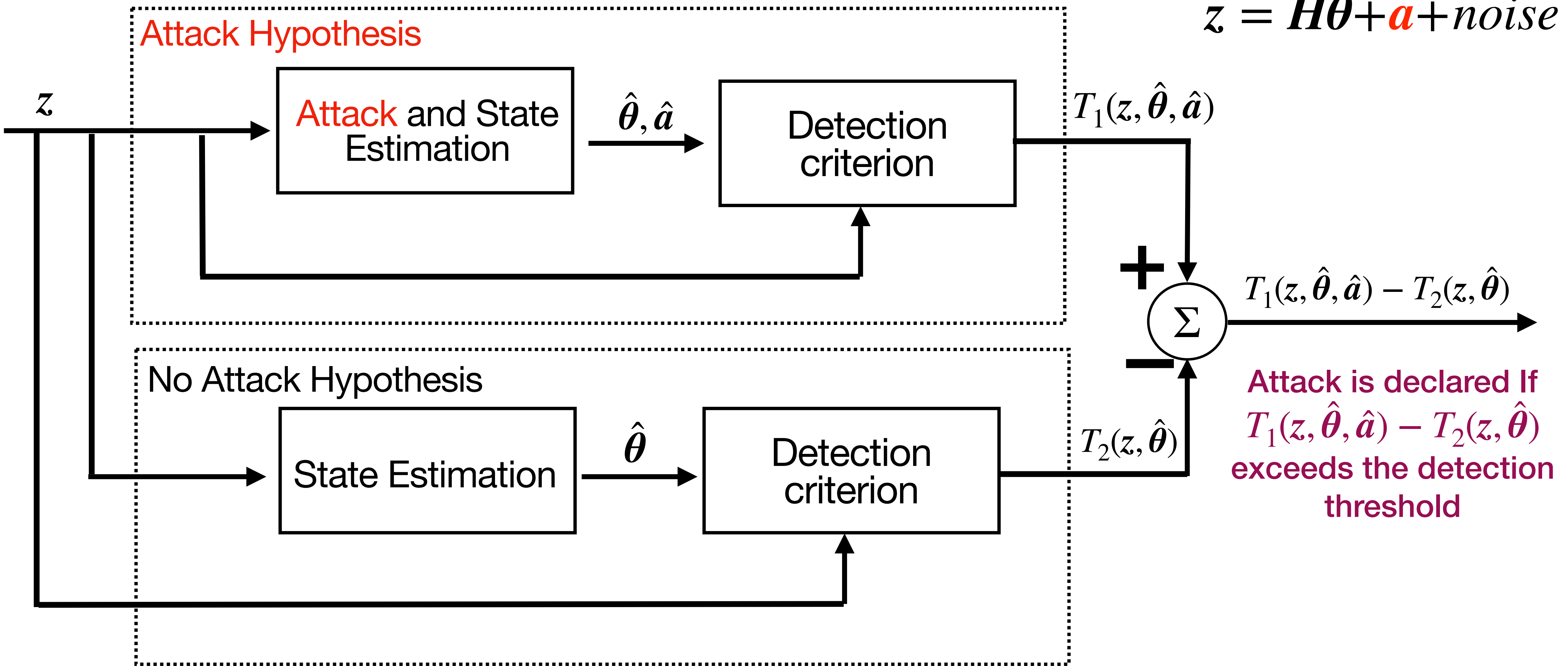
# Secured Sensors and Graph Based Detection

$$z = H\theta + a + noise$$



# Secured Sensors and Graph Based Detection

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# Secured Sensors and Graph Based Detection

## Attack Hypothesis

$$T_1(\mathbf{z}, \boldsymbol{\theta}, \mathbf{a}) = \|\mathbf{z} - \mathbf{H}\boldsymbol{\theta} - \mathbf{a}\|_2^2 - \mu_1 \|\mathbf{a}_S\|_2^2 - \mu_2 \sum_{v \in \mathcal{V}} \sum_{u \in \mathcal{N}_v} w_{v,u} (\theta_u - \theta_v)^2$$

$$\hat{\boldsymbol{\theta}} = \min_{\boldsymbol{\theta}, \mathbf{a}} T_1(\mathbf{z}, \boldsymbol{\theta}, \mathbf{a})$$

## No Attack Hypothesis

$$T_2(\mathbf{z}, \boldsymbol{\theta}) = \|\mathbf{z} - \mathbf{H}\boldsymbol{\theta}\|_2^2 - \mu_2 \sum_{v \in \mathcal{V}} \sum_{u \in \mathcal{N}_v} \omega_{v,u} (\theta_u - \theta_v)^2$$

$$\hat{\boldsymbol{\theta}} = \min_{\boldsymbol{\theta}} T_2(\mathbf{z}, \boldsymbol{\theta})$$

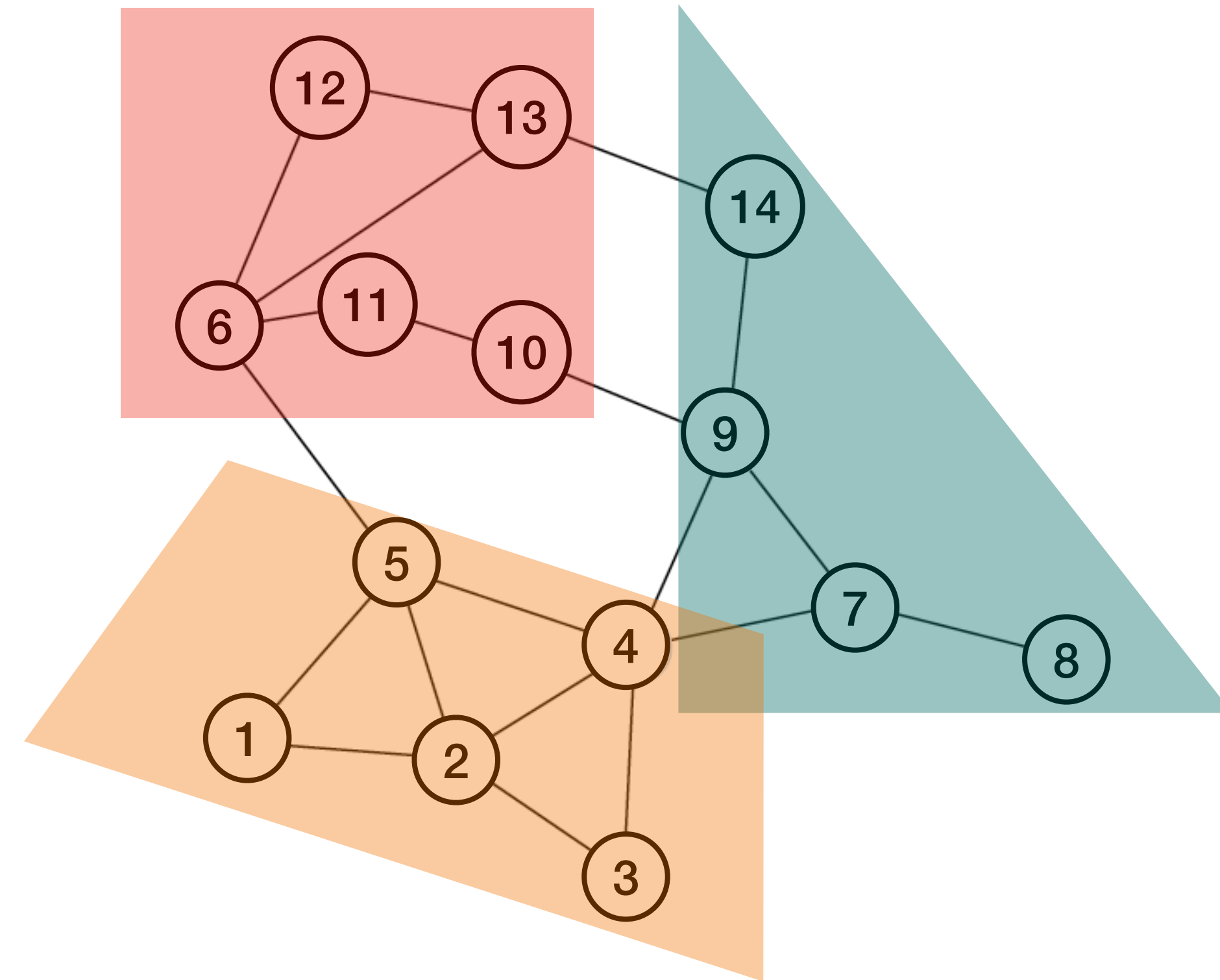
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- **Theory: Modification for Distributed Optimization**
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# Modification to Distributed Optimization

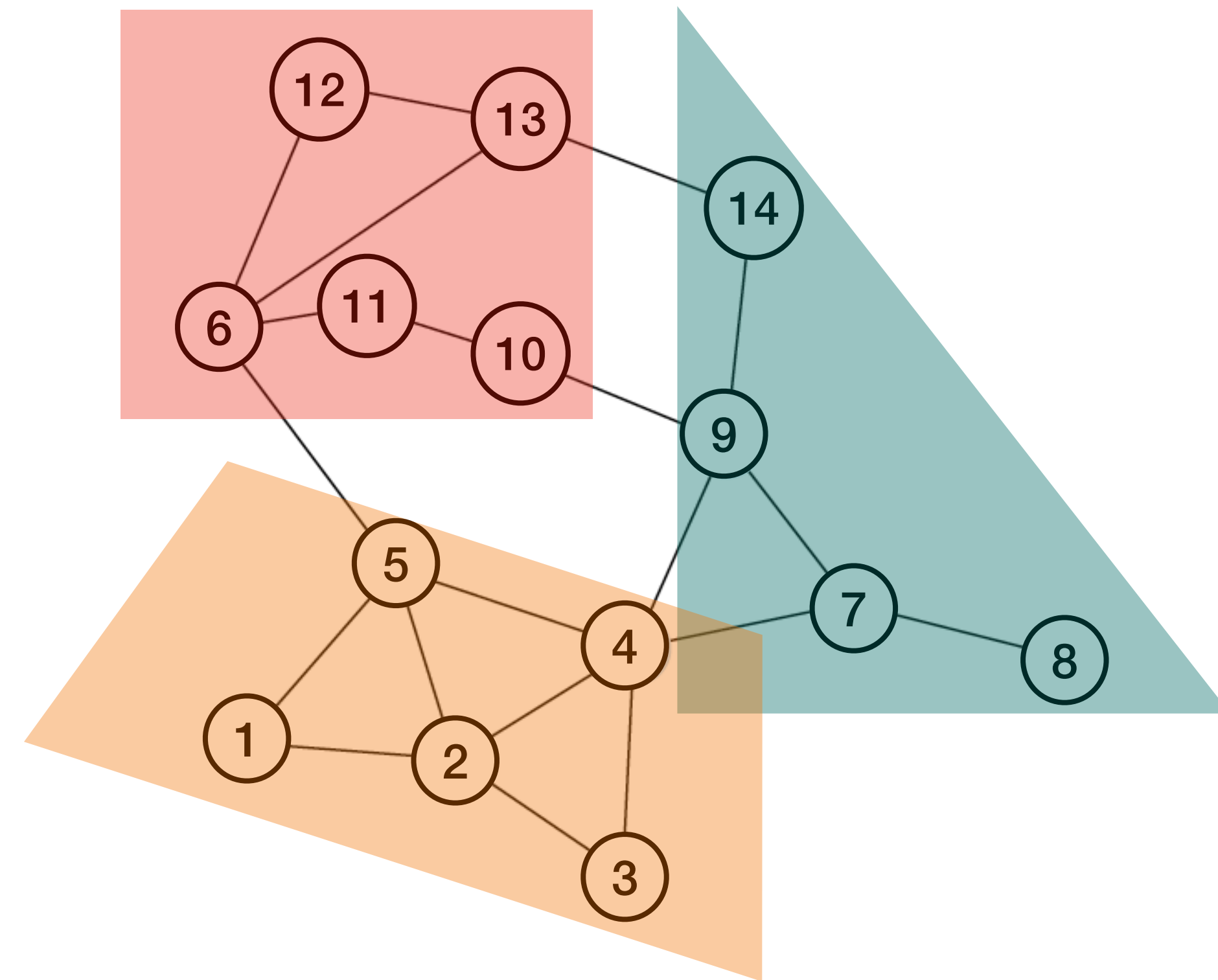
$$z_l = H_l \theta_l + a_l + \text{noise}, \quad l = 1, 2, \dots$$



# Modification to Distributed Optimization

$$z_l = H_l \theta_l + a_l + noise, \quad l = 1, 2, \dots$$

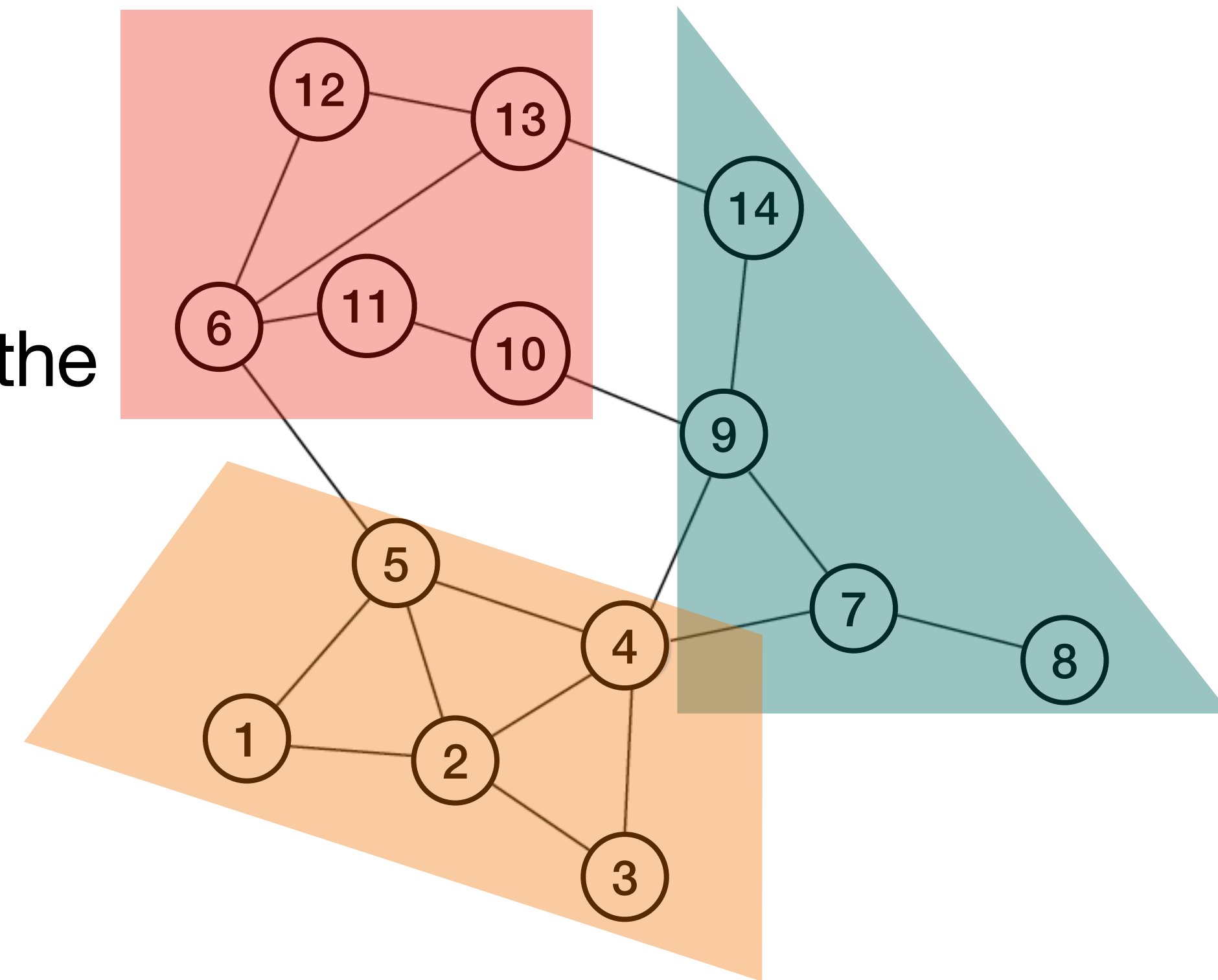
- The measurements in each area are contained in the area  
e.g. vertex measurement in the orange area are: 1,2,3,4,5



# Modification to Distributed Optimization

$$z_l = H_l \theta_l + a_l + noise, \quad l = 1, 2, \dots$$

- The measurements in each area are contained in the area  
e.g. vertex measurement in the orange area are: 1,2,3,4,5
- The state variables in each area are the ones contained in the area and their first order neighbors.  
e.g. state variables in the orange area are: 1,2,3,4,5,6,7,9

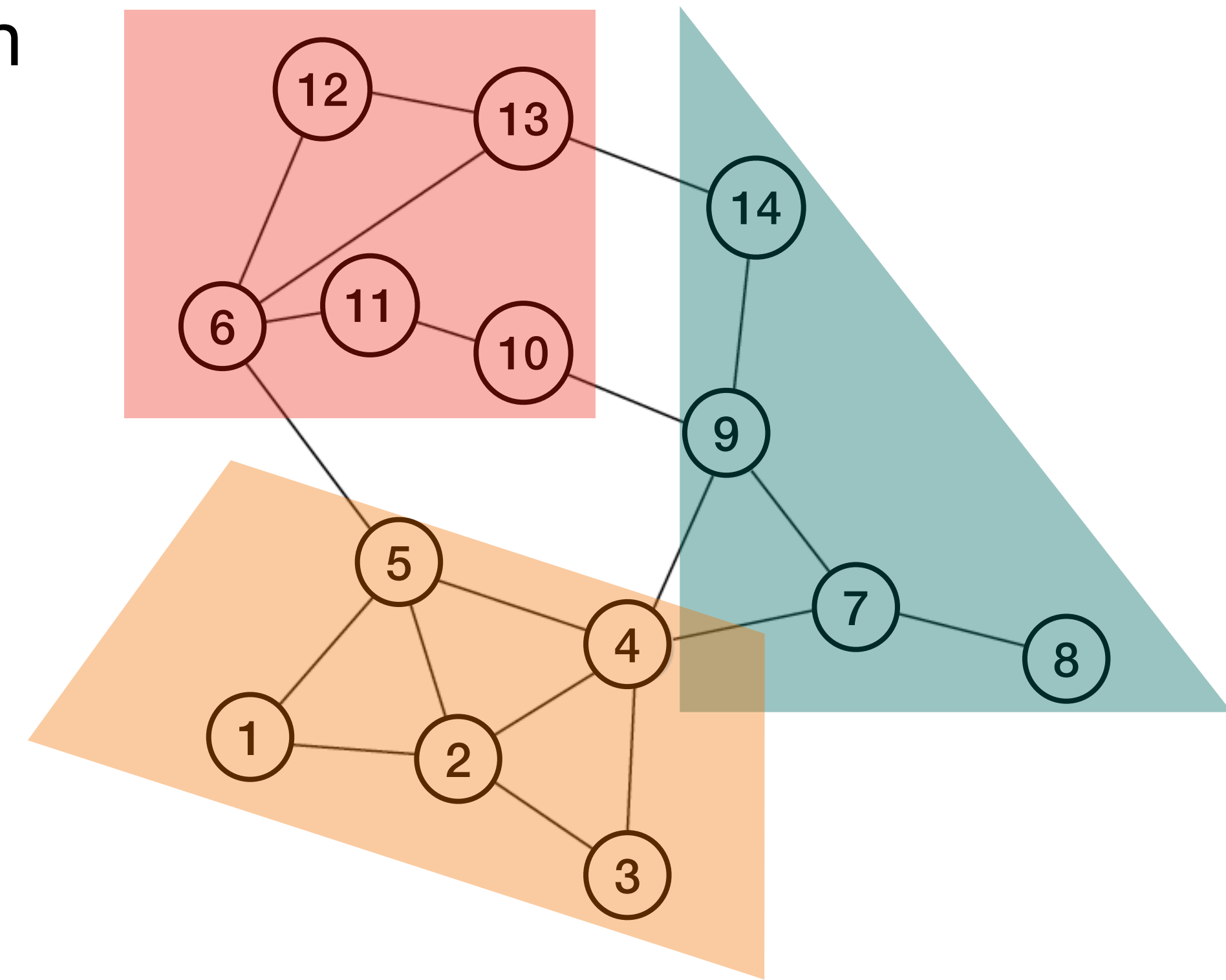


There is an overlap between  
state variables in neighbor areas

# Modification to Distributed Optimization

$$z_l = H_l \theta_l + a_l + noise, \quad l = 1, 2, \dots$$

- State estimation and attack detection is performed in each area separately

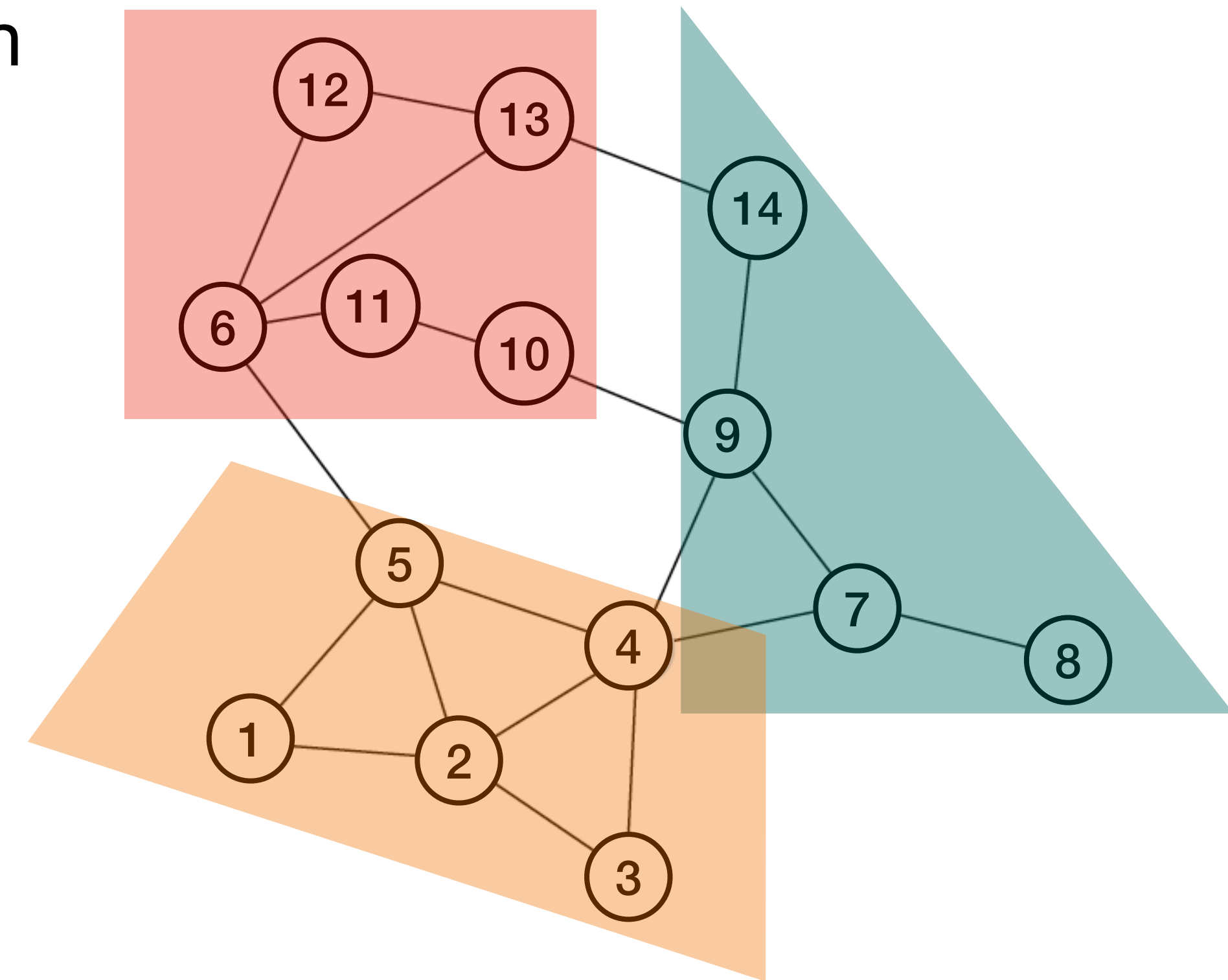




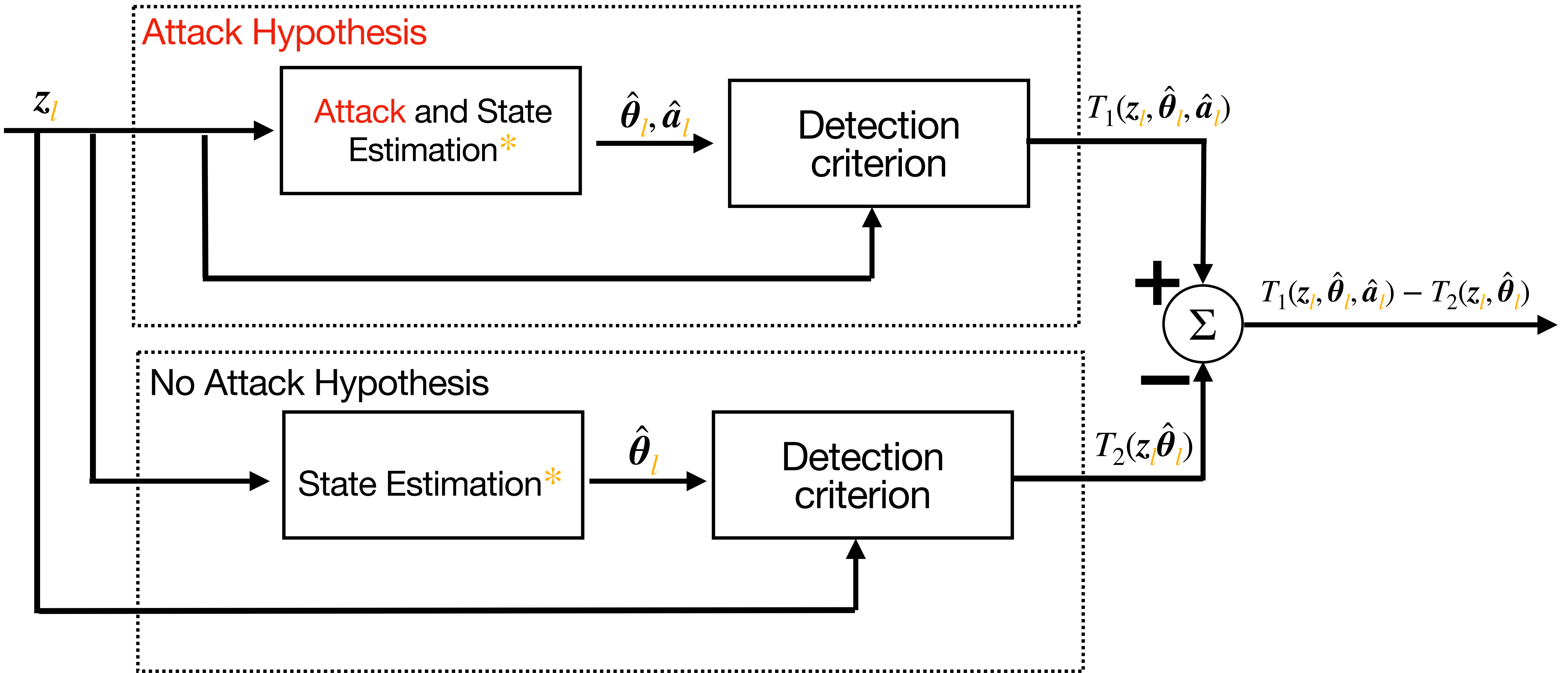
# Modification to Distributed Optimization

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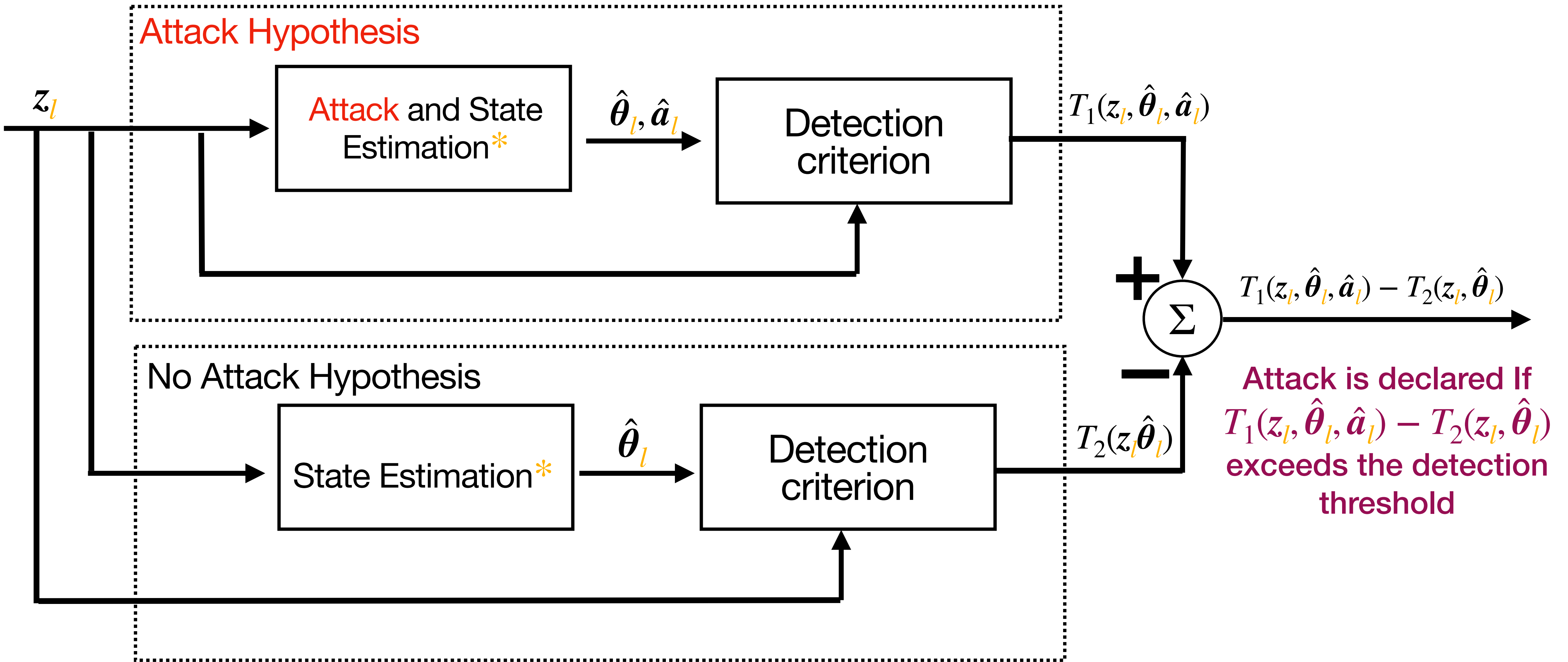
- State estimation and attack detection is performed in each area separately
- Estimation (states and attack) is performed in each area iteratively
- In each iteration, the control centers in neighbor areas share information on their state variables



# Secured Sensors and Graph Based Detection



# Secured Sensors and Graph Based Detection



# Attack and State Estimation

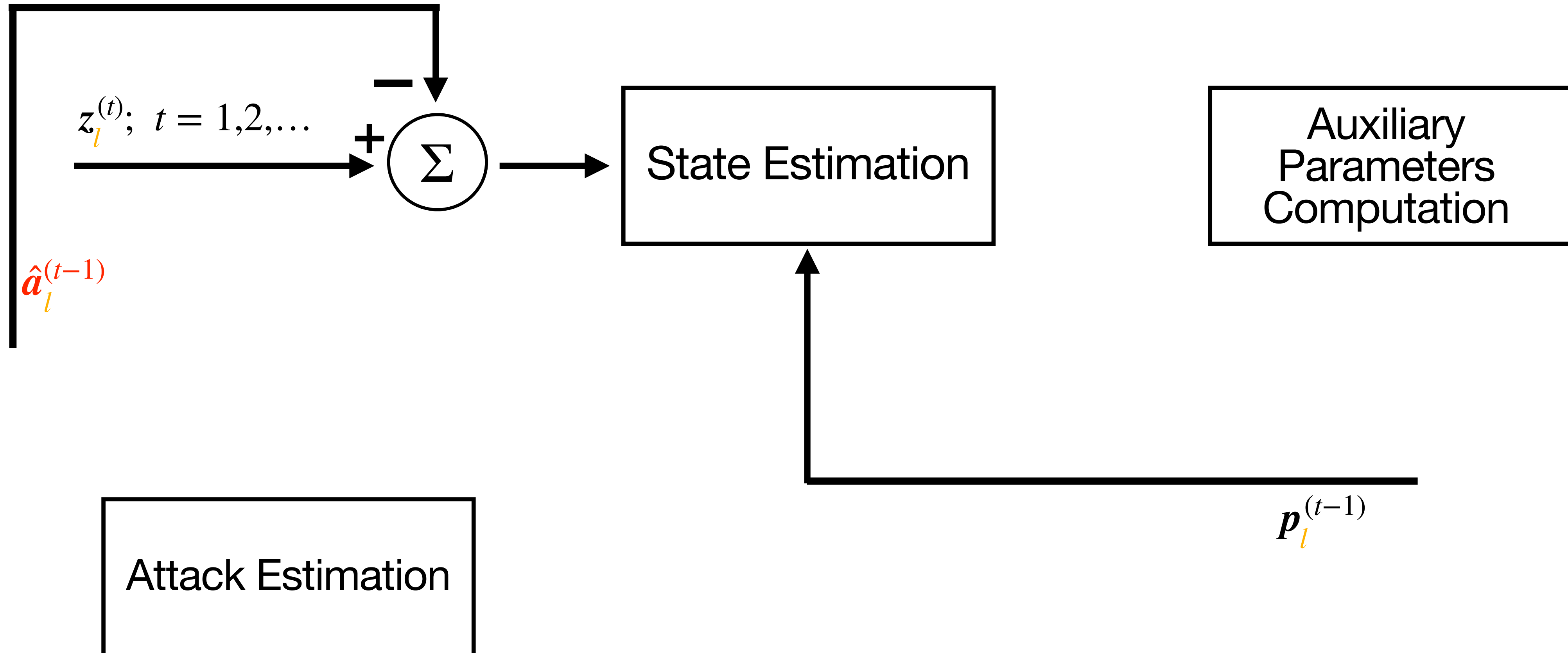
State Estimation

Auxiliary  
Parameters  
Computation

Attack Estimation

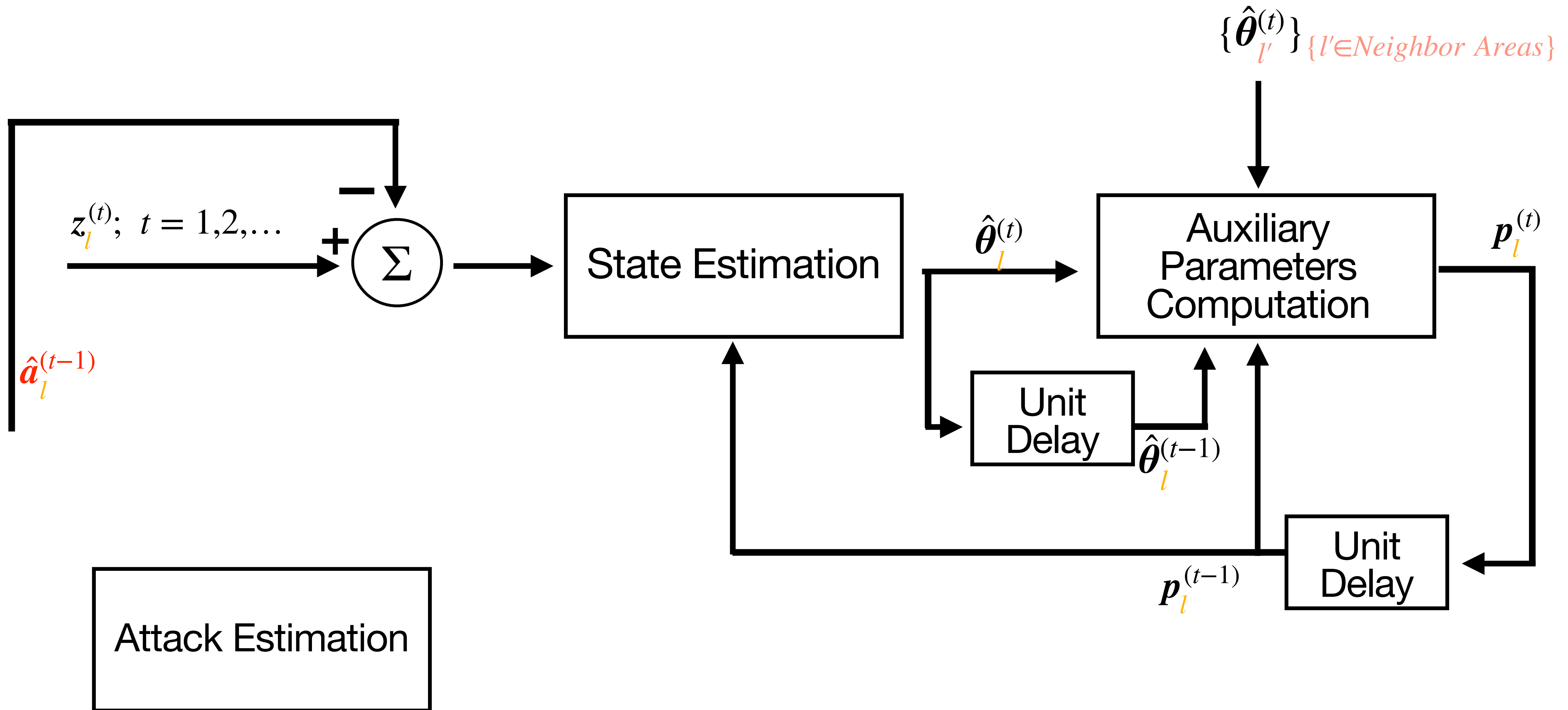
\* Explained for the Attack hypothesis

# Attack and State Estimation



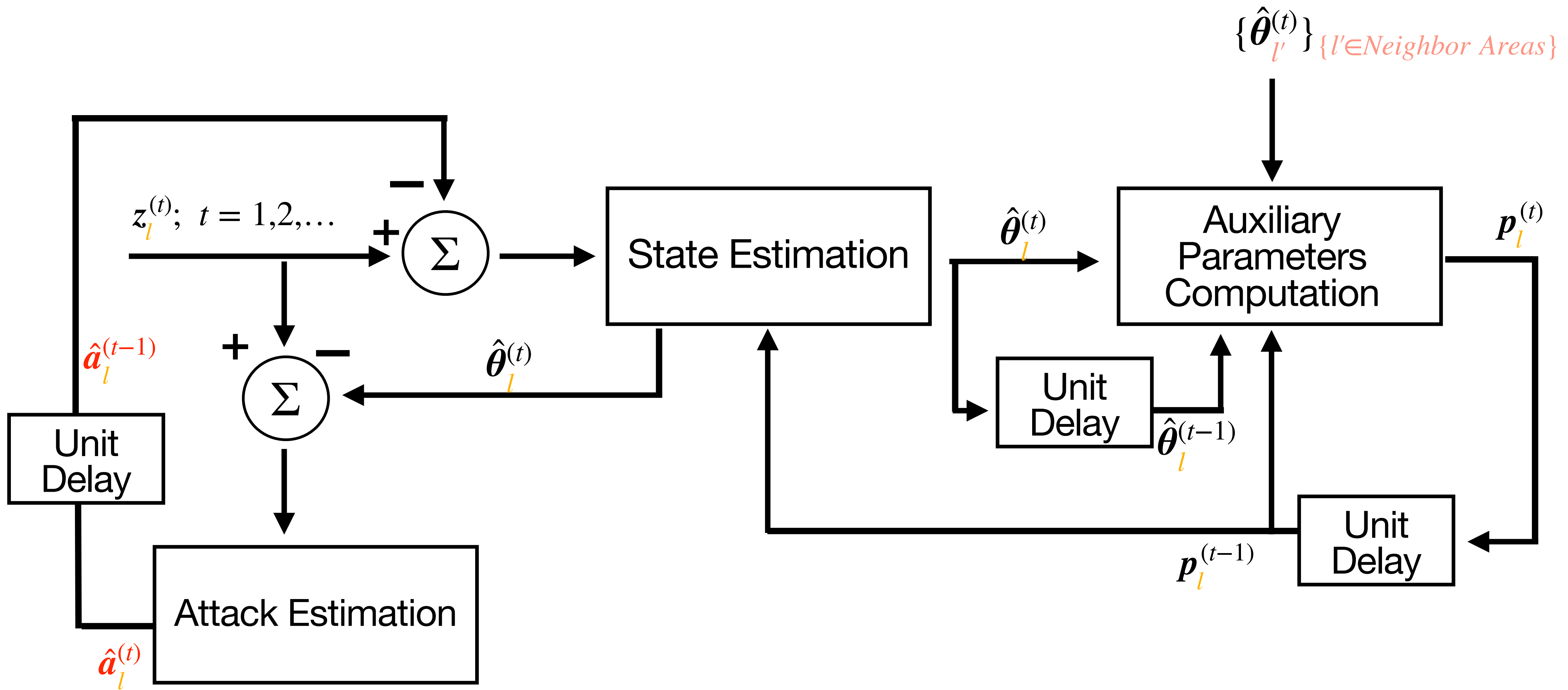
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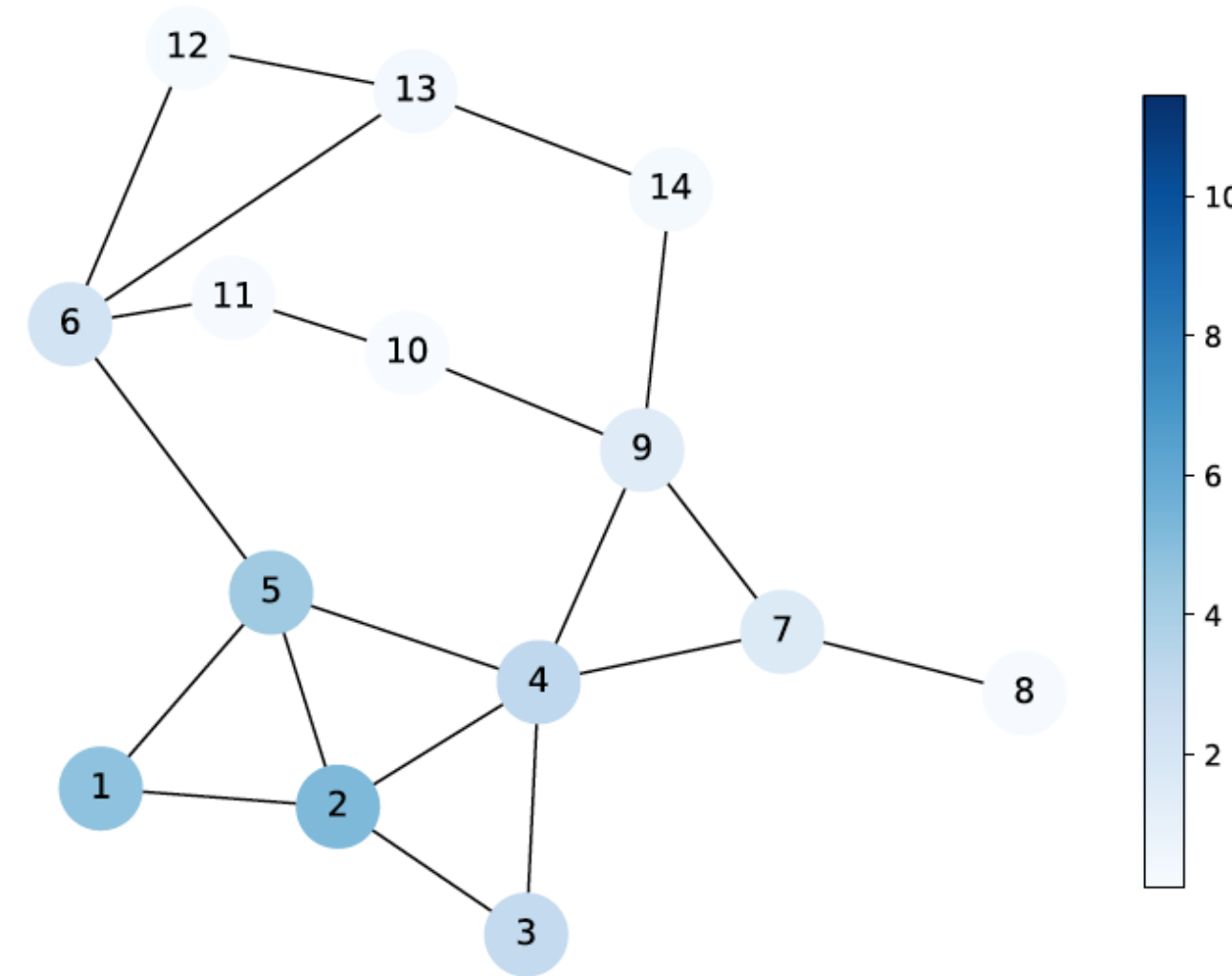
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# Power System States are Low Pass Graph Signals

The difference between the signal state values in neighbor vertices is assumed small



## State Signal

Each vertex is assigned with a value represented by its color

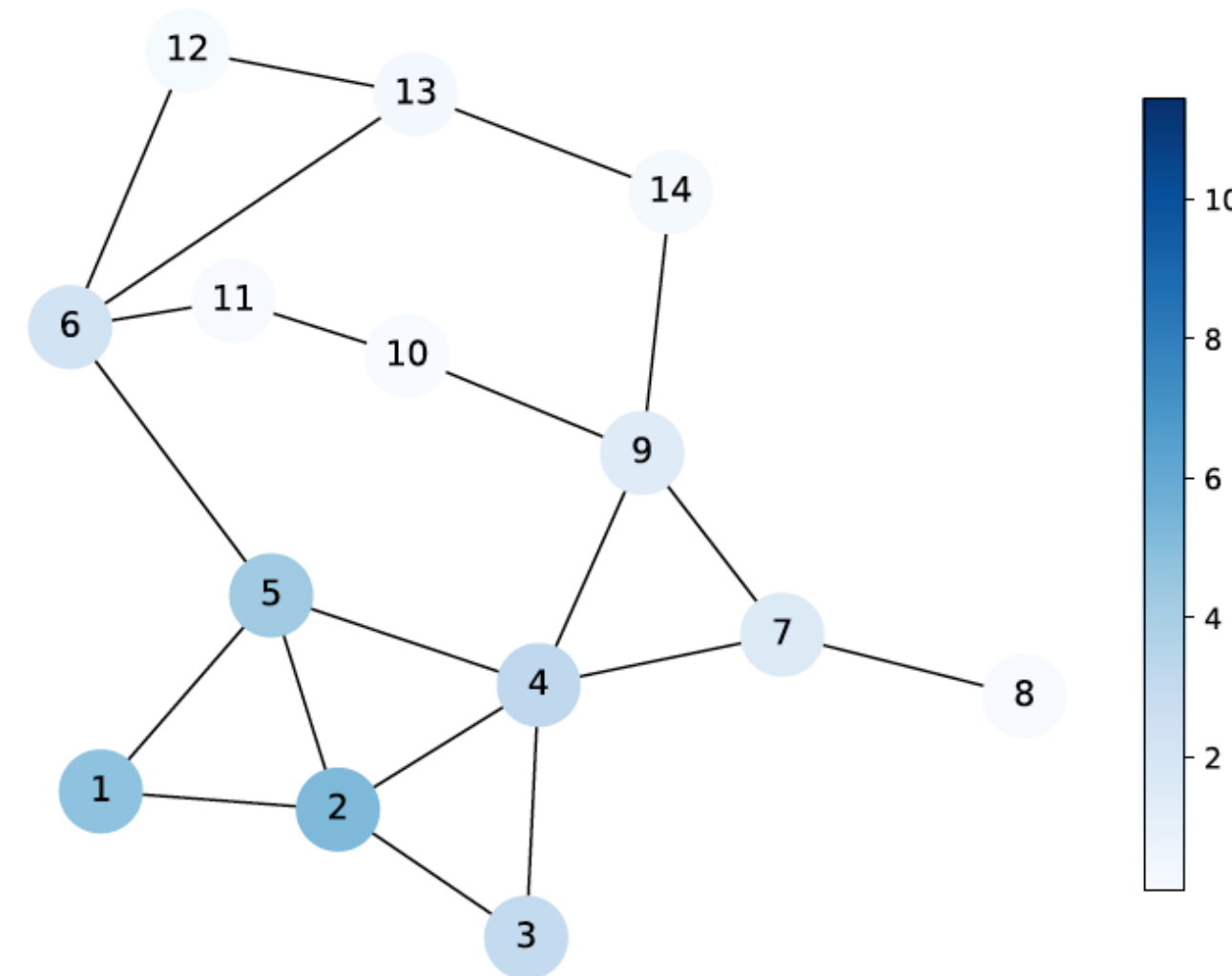
Drayer, Elisabeth, and Tirza Routtenberg. "Detection of false data injection attacks in smart grids based on graph signal processing." *IEEE Systems Journal* 14.2 (2019): 1886-1896.

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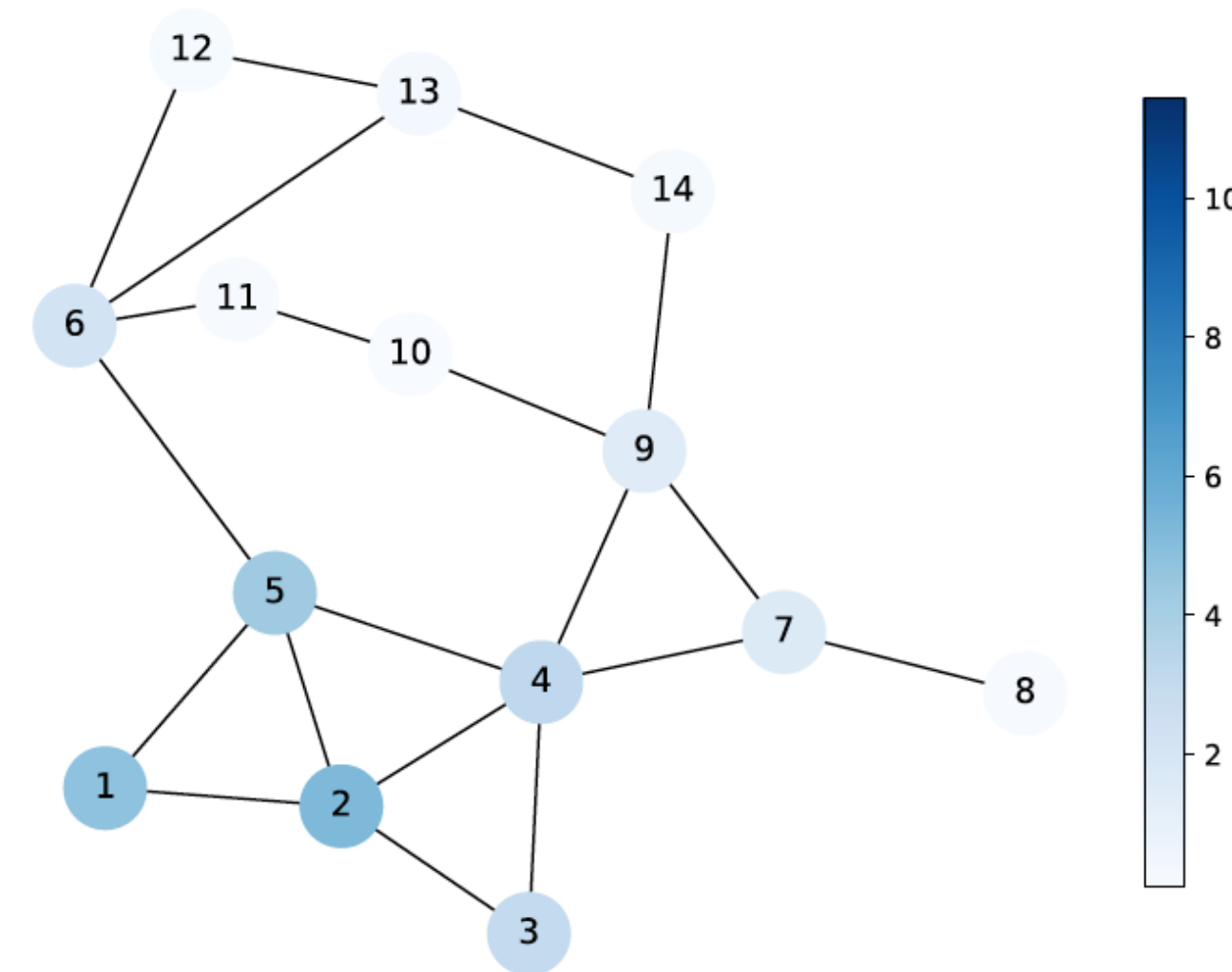
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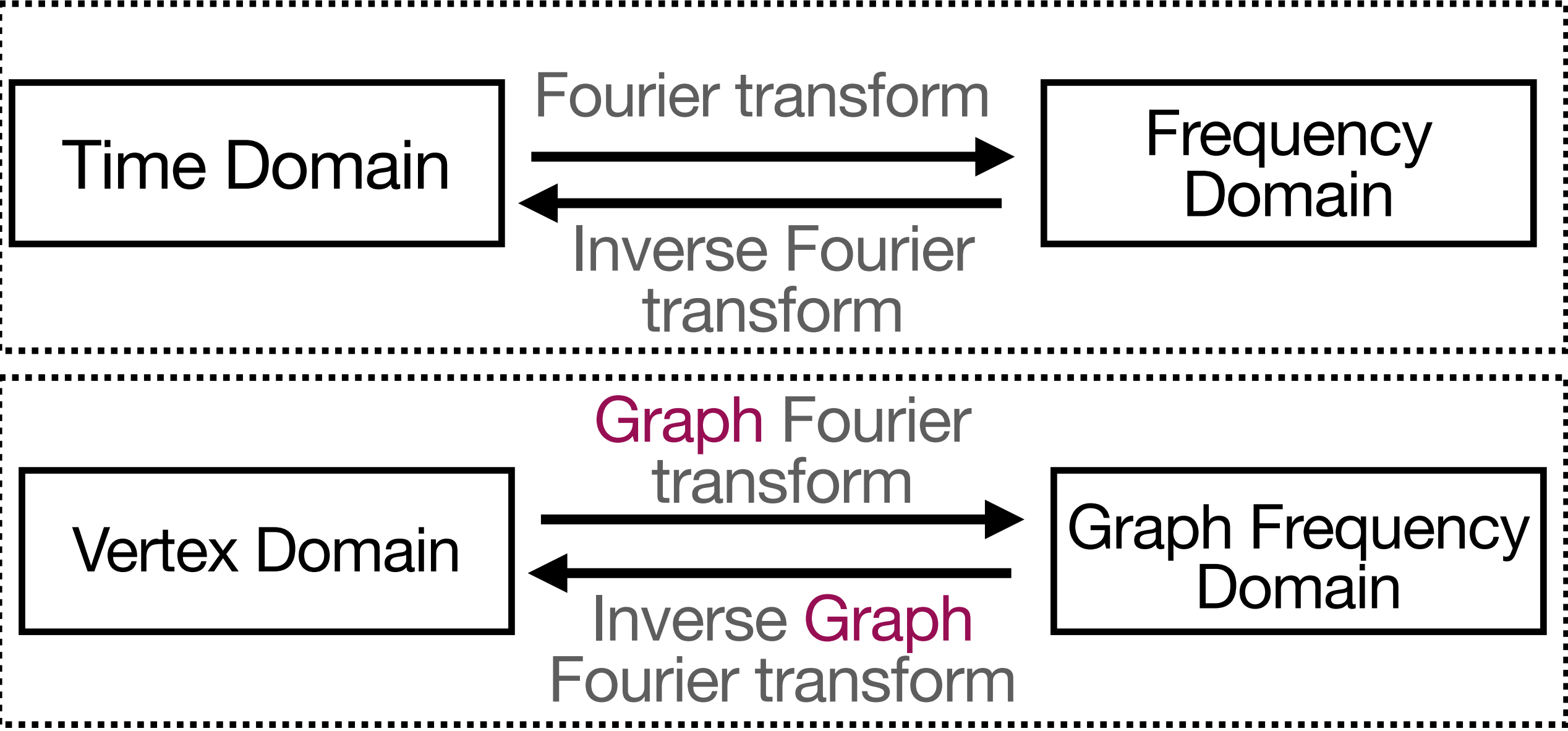
The signals are assumed to be low pass graph signals



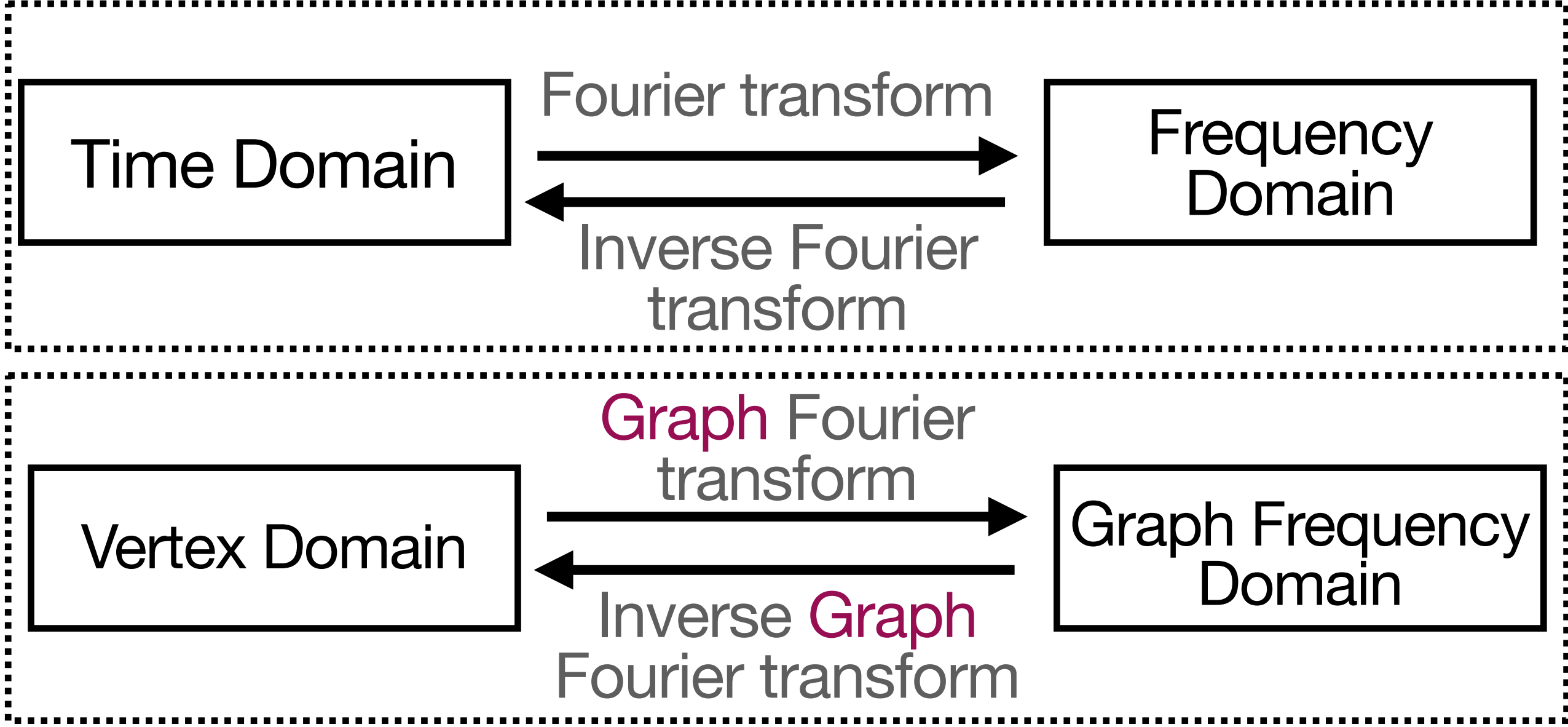
## State Signal

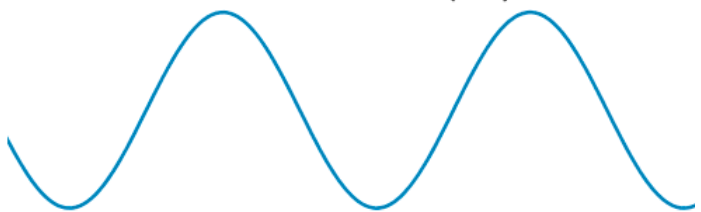
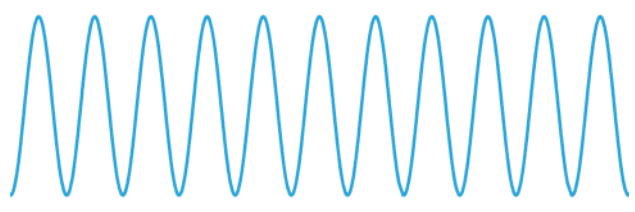
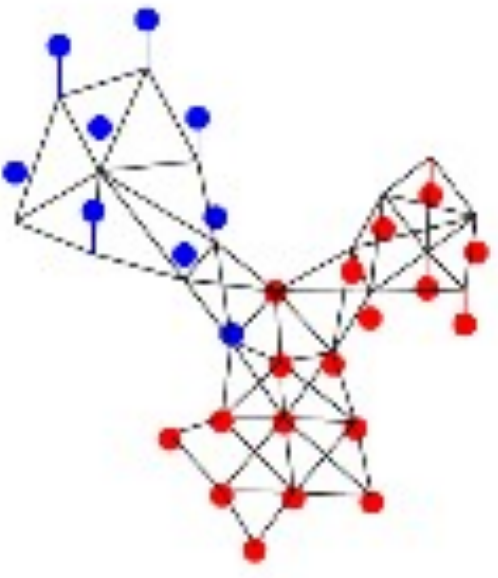
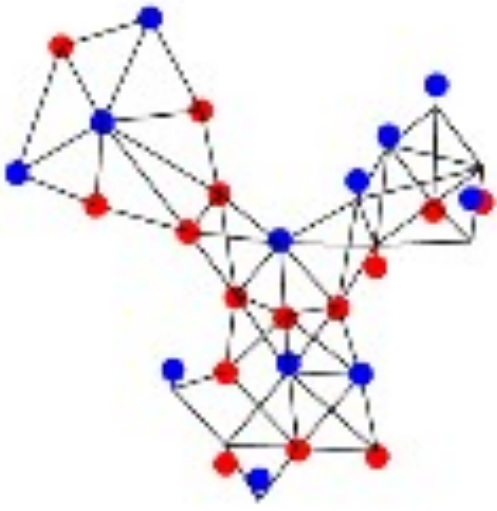
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# Power System States are Low Pass Graph Signals



# Power System States are Low Pass Graph Signals



	Low Frequency	High Frequency
Time Signals		
Graph Signals		

high value/low value

# Power System States are Low Pass Graph Signals

Low graph total variation

$$\sum_{v \in \mathcal{V}} \sum_{u \in \mathcal{N}_v} w_{v,u} (\theta_u - \theta_v)^2 \leq \epsilon_2$$

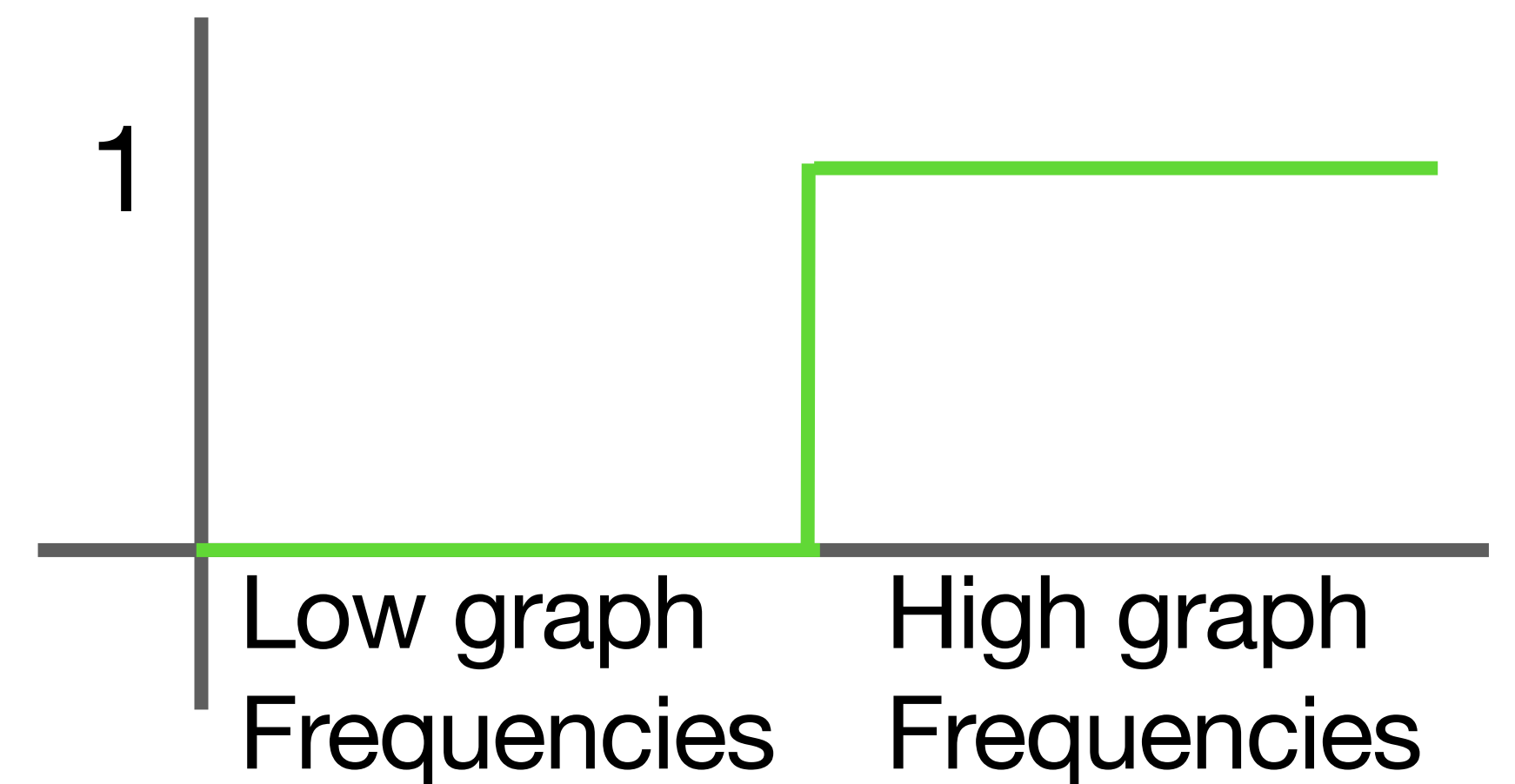


Low energy in high graph frequencies

$$\|F\theta\|_2^2 \leq \epsilon_2$$

$F$  - graph high pass filter

Ideal Graph High Pass Filter ( $F$ )



# Power System States are Low Pass Graph Signals

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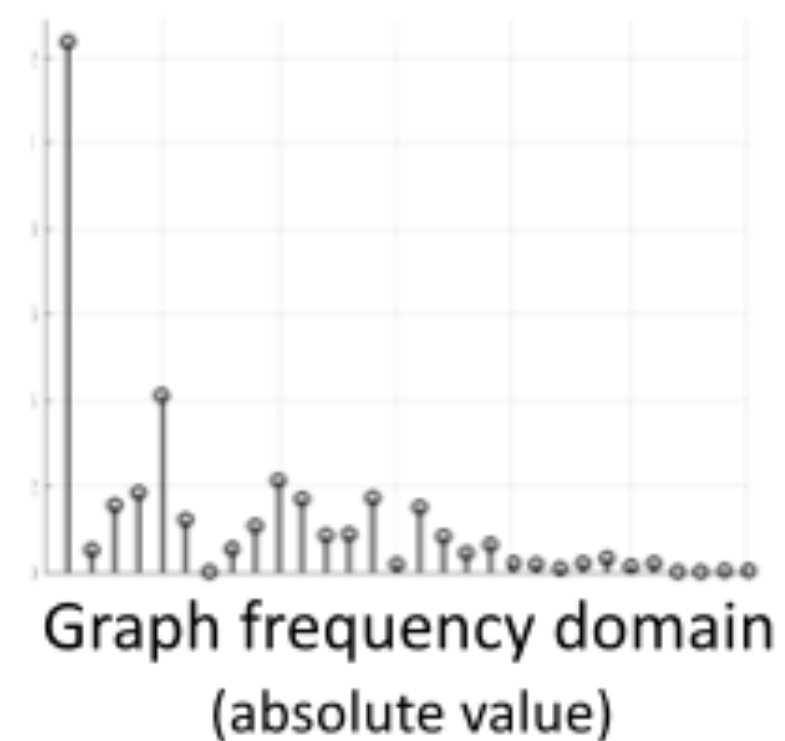
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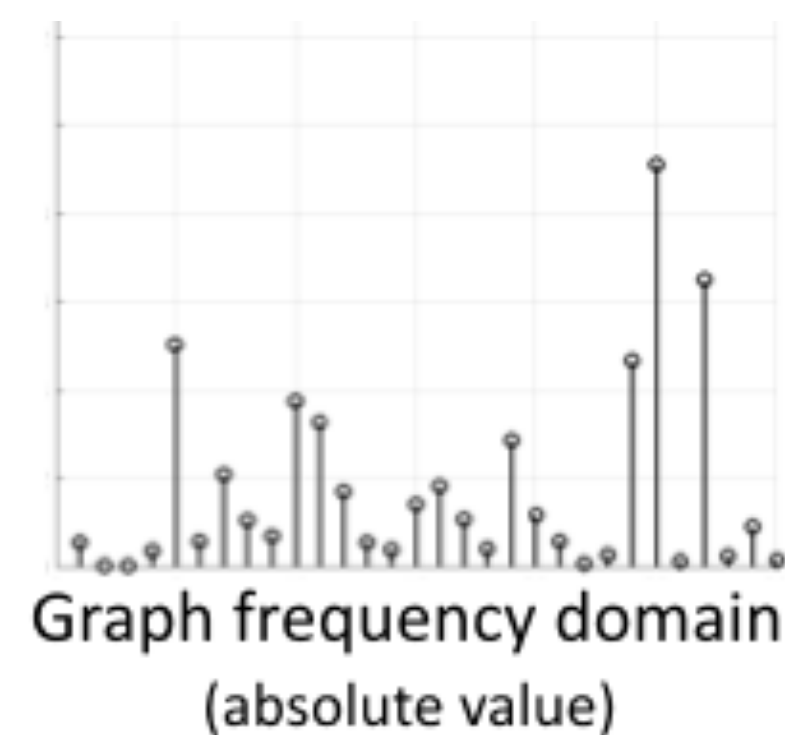
$F$  - graph high pass filter

Example

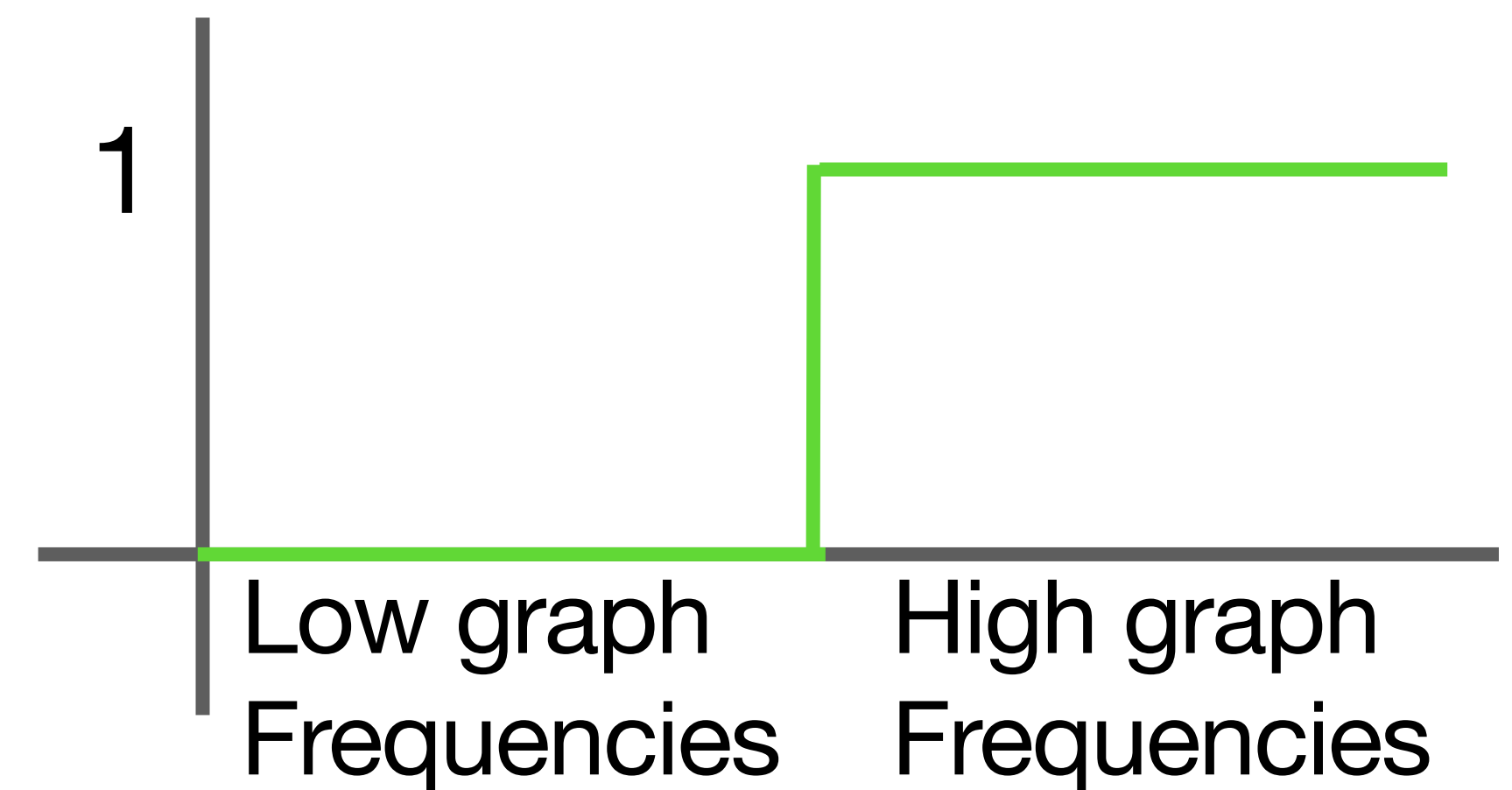
state signal  
(low pass)



attack signal



Ideal Graph High Pass Filter ( $F$ )



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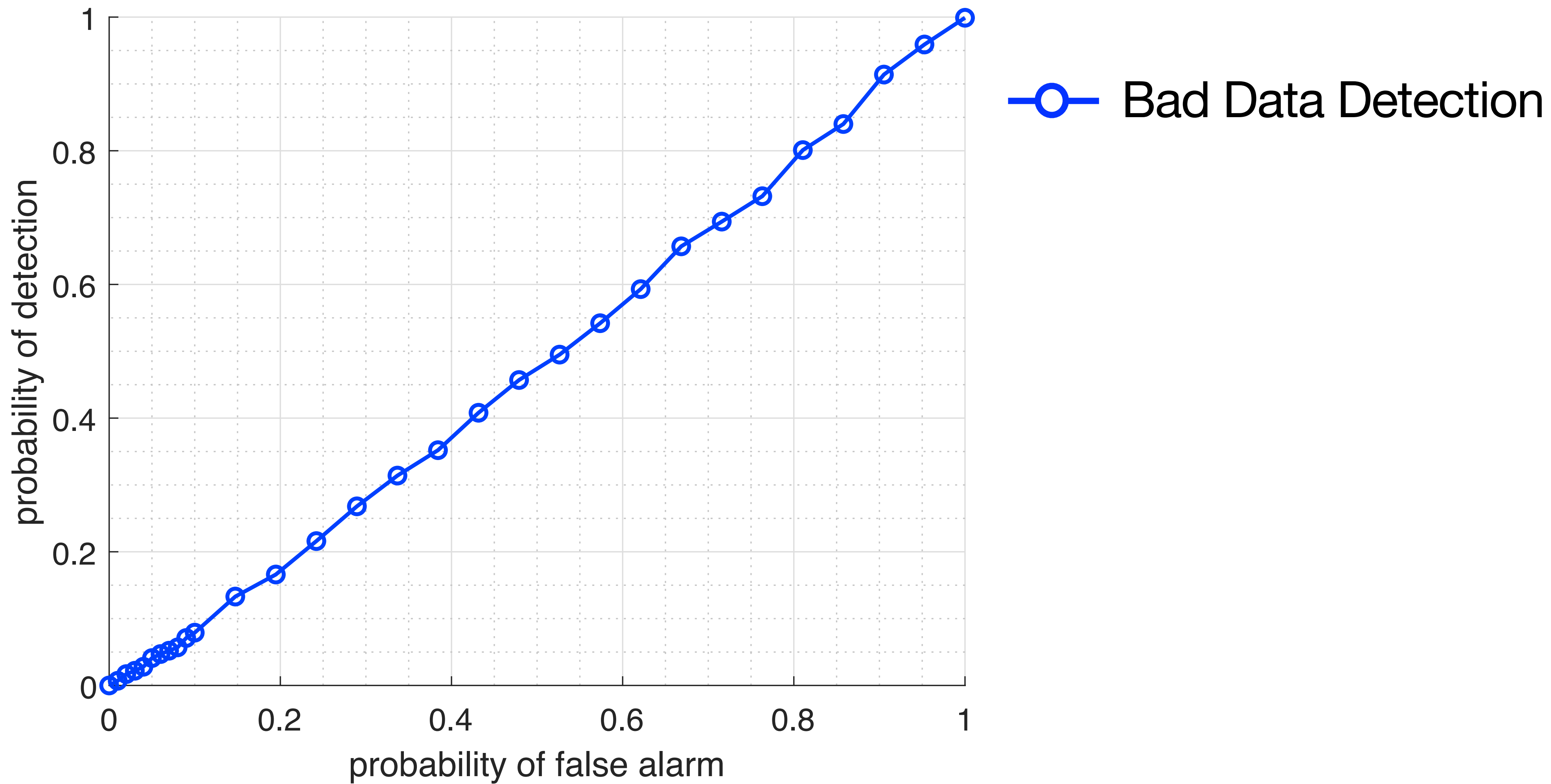


# Performance Evaluation

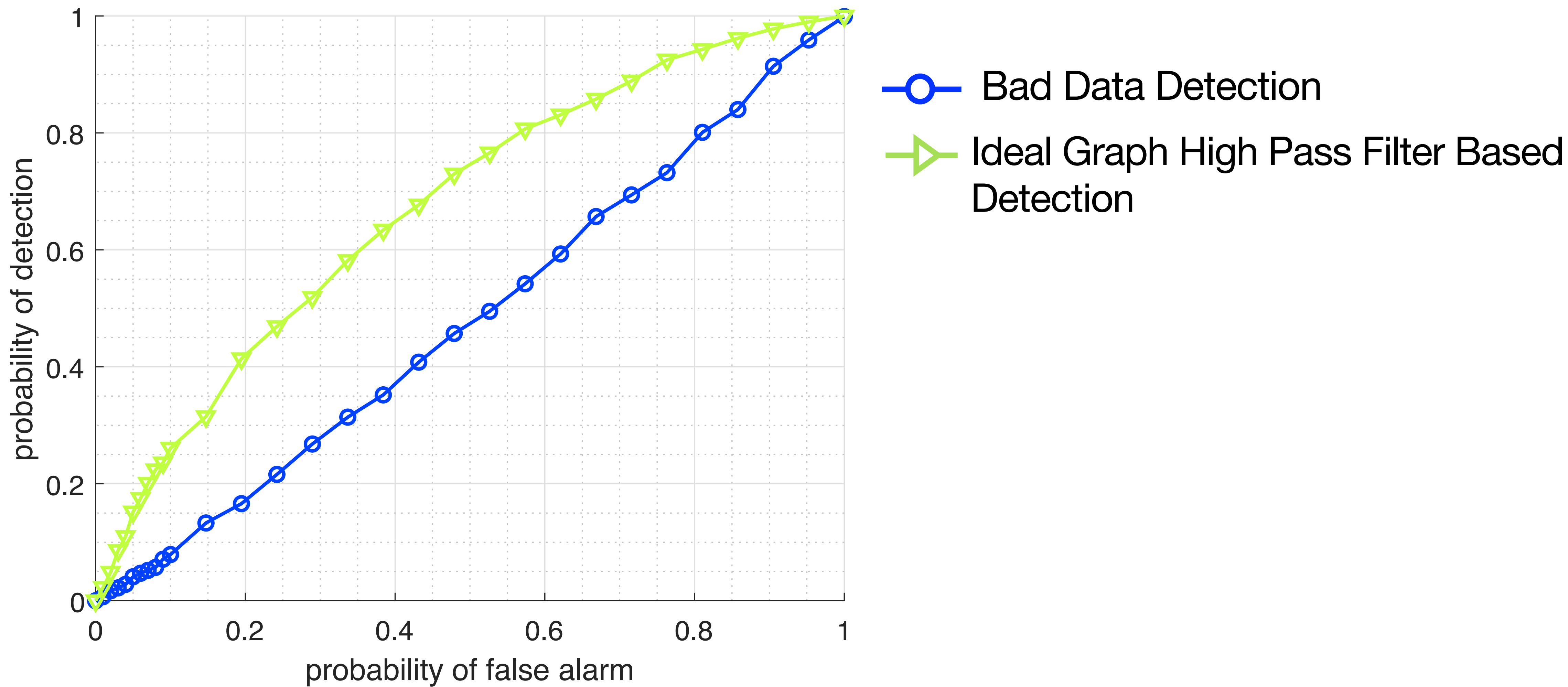
## Set up

- IEEE-57 power system test case
- Attack on a single node (33)
- 36% of the measurements are secured
  - Ensuring the state variables in the generator substations cannot be manipulated

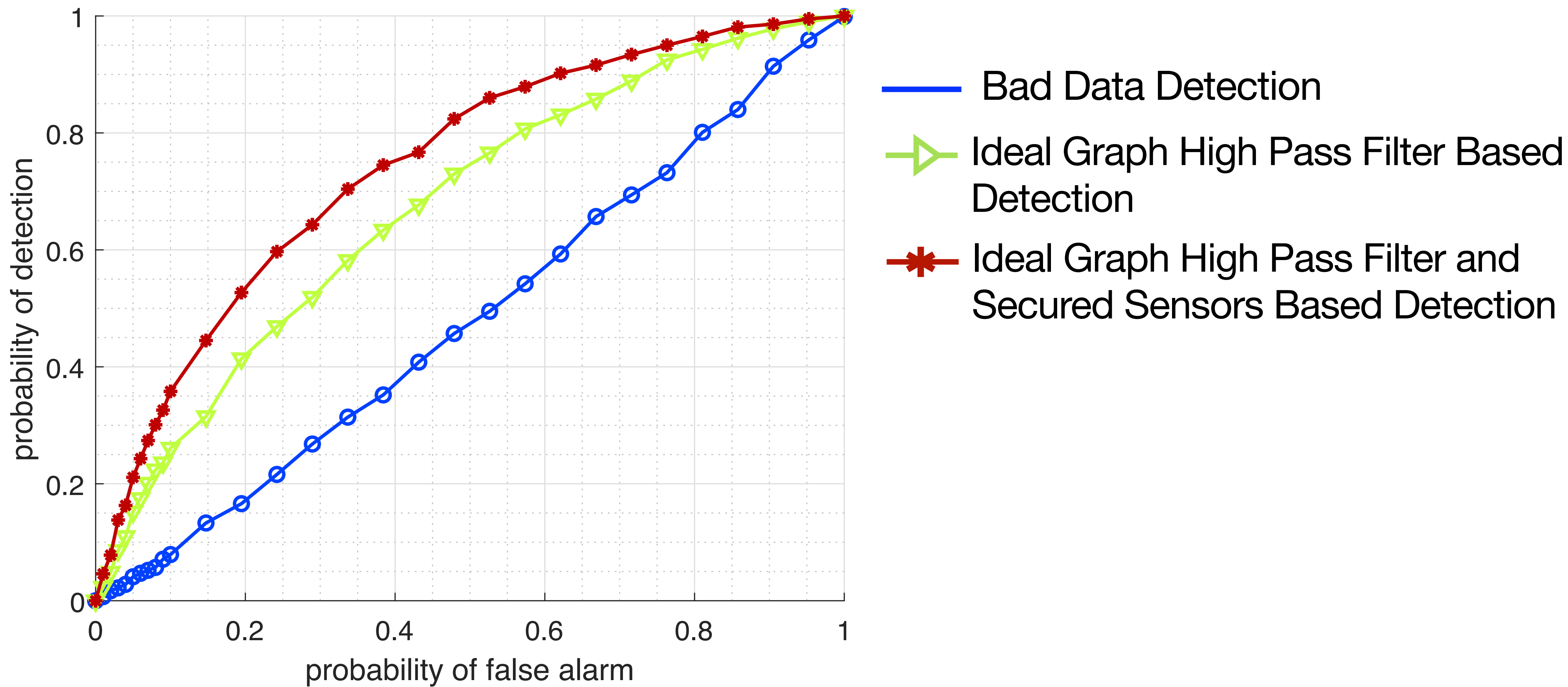
# Receiver Operating Characteristics (ROC)



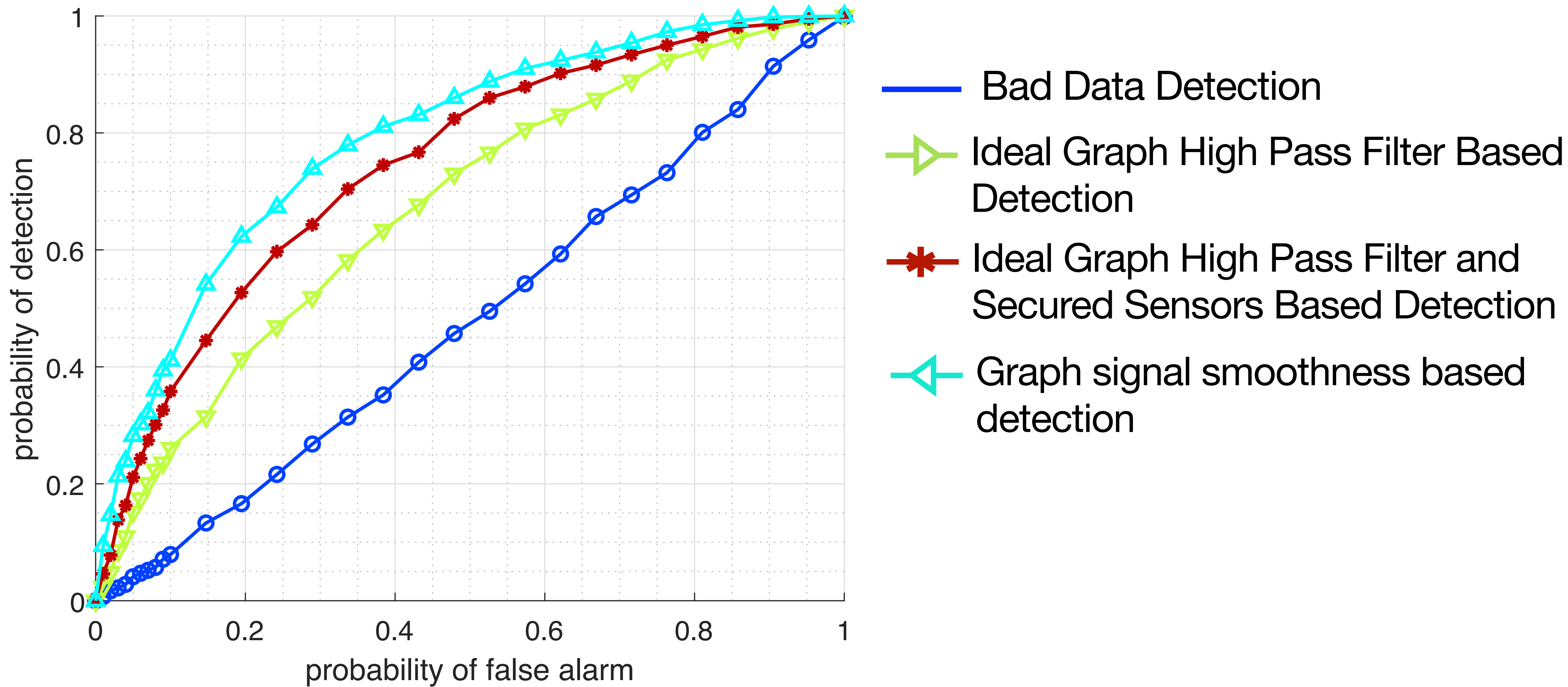
# Receiver Operating Characteristics (ROC)



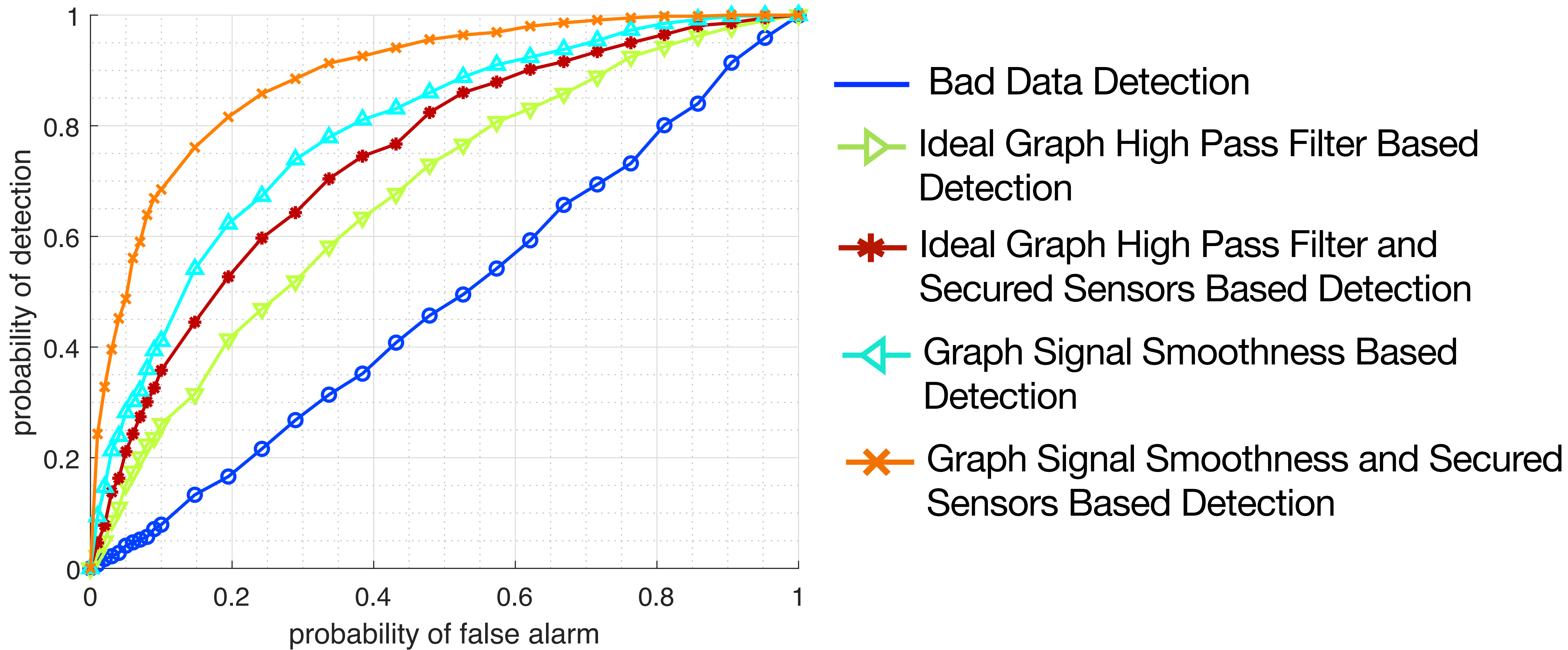
# Receiver Operating Characteristics (ROC)



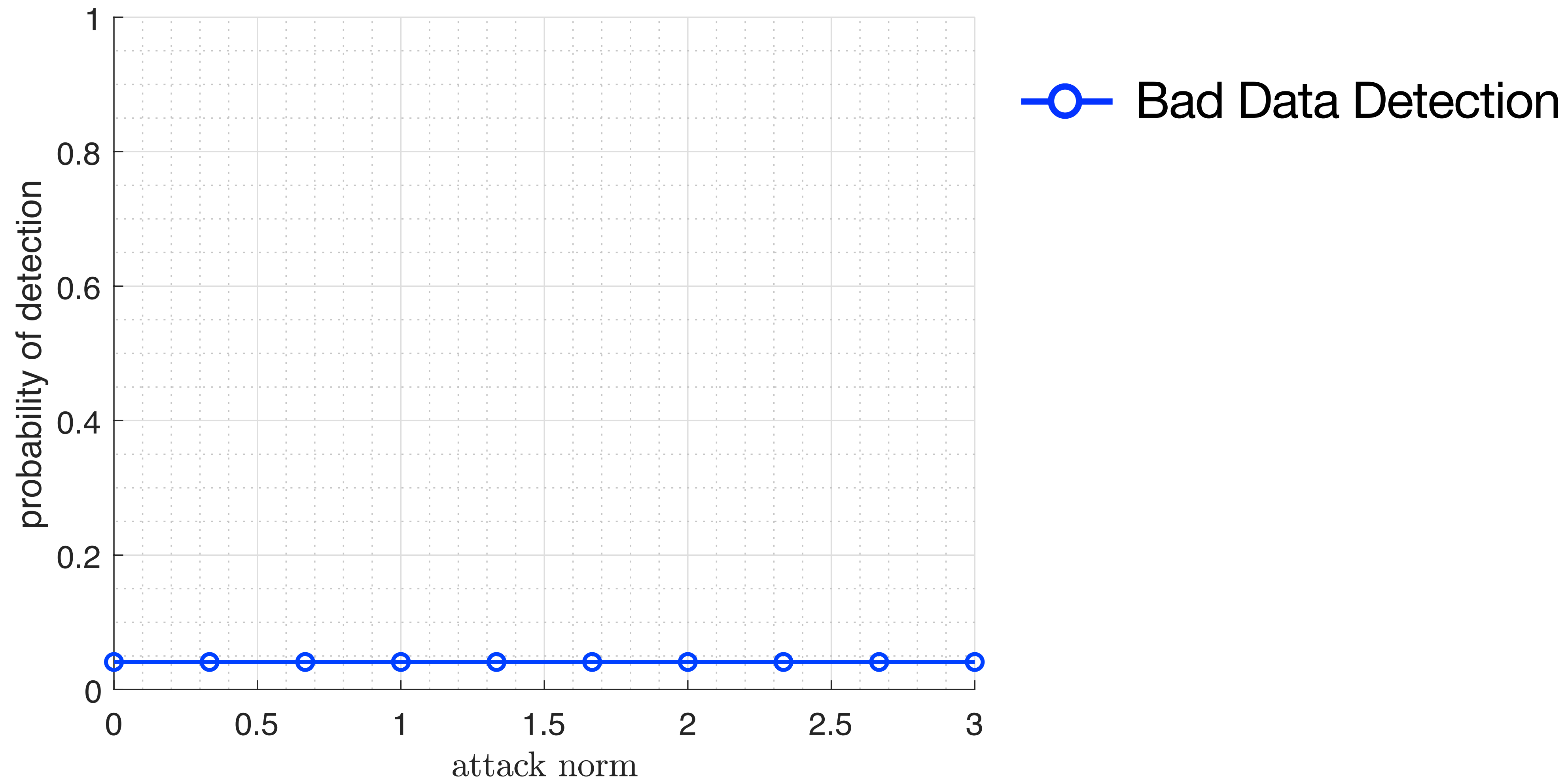
# Receiver Operating Characteristics (ROC)



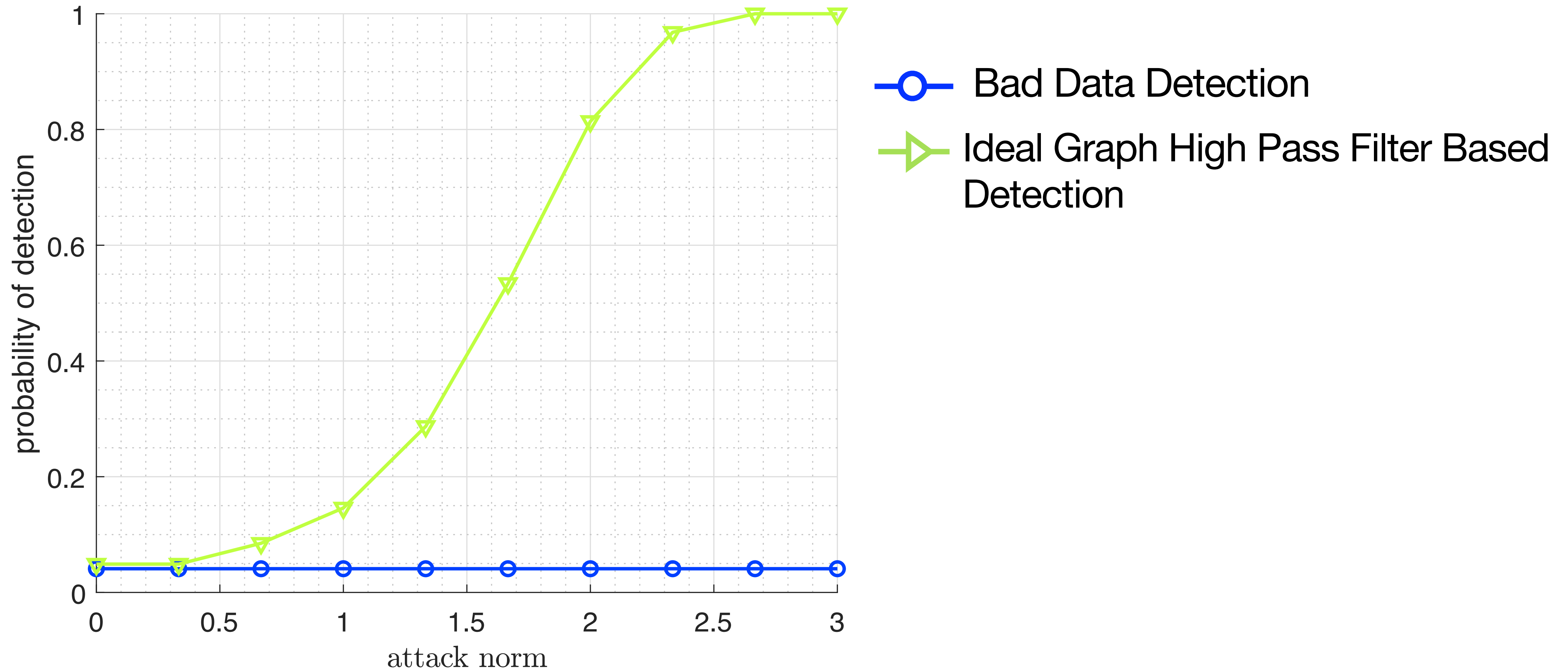
# Receiver Operating Characteristics (ROC)



# Probability of Detection Verse Attack Norm

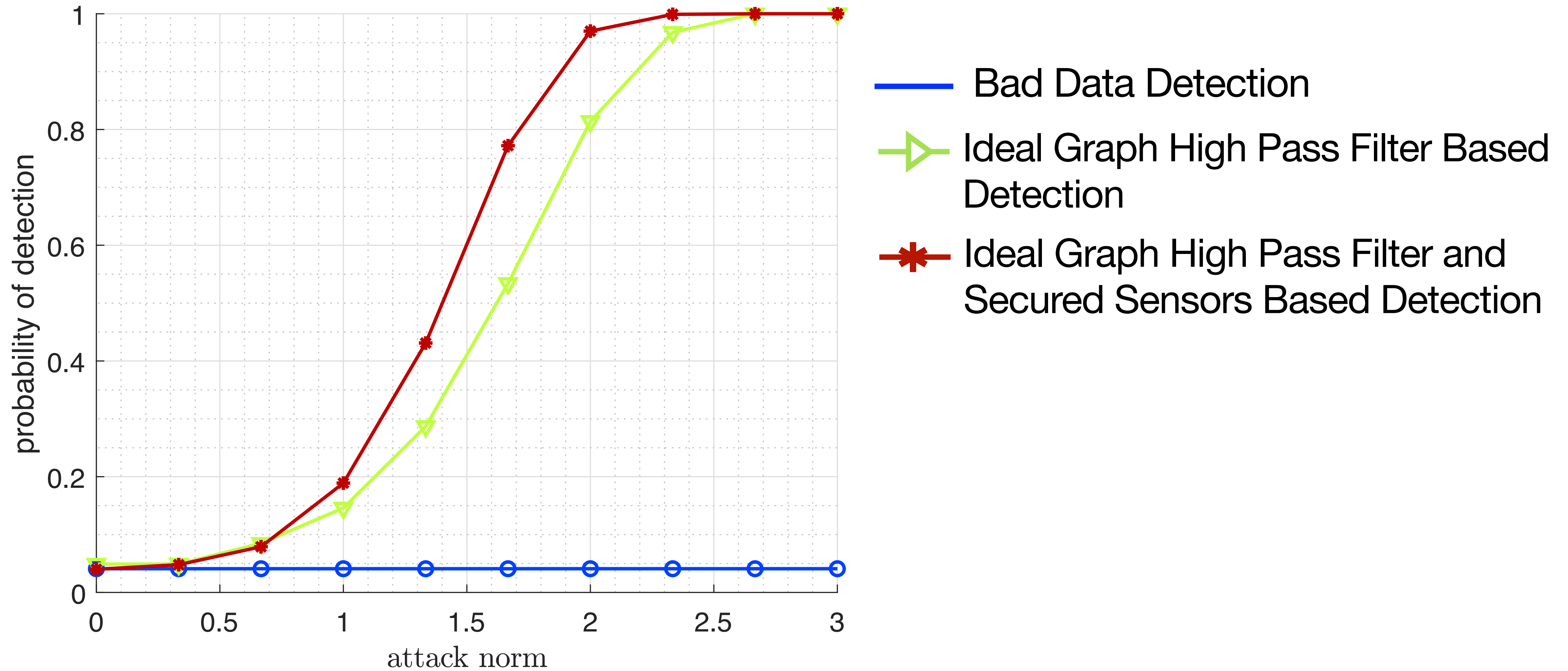


# Probability of Detection Verse Attack Norm

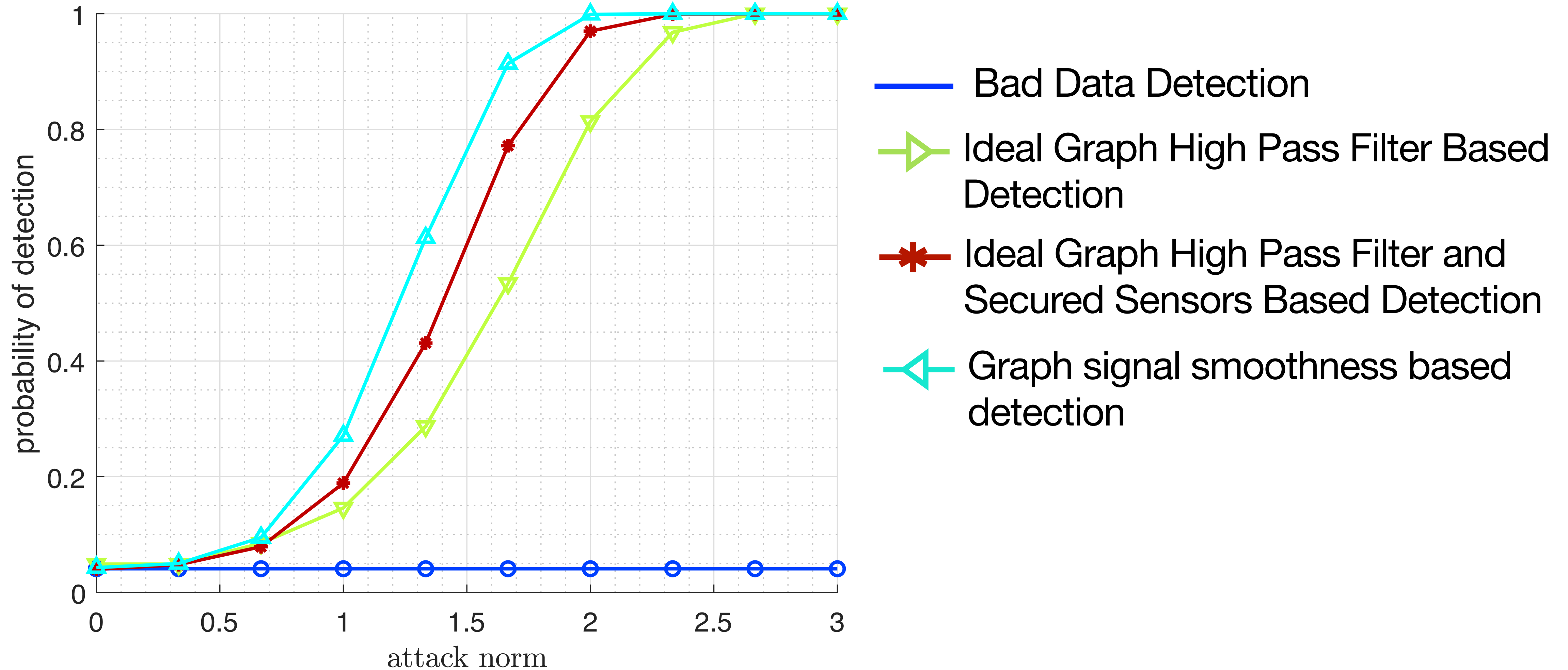




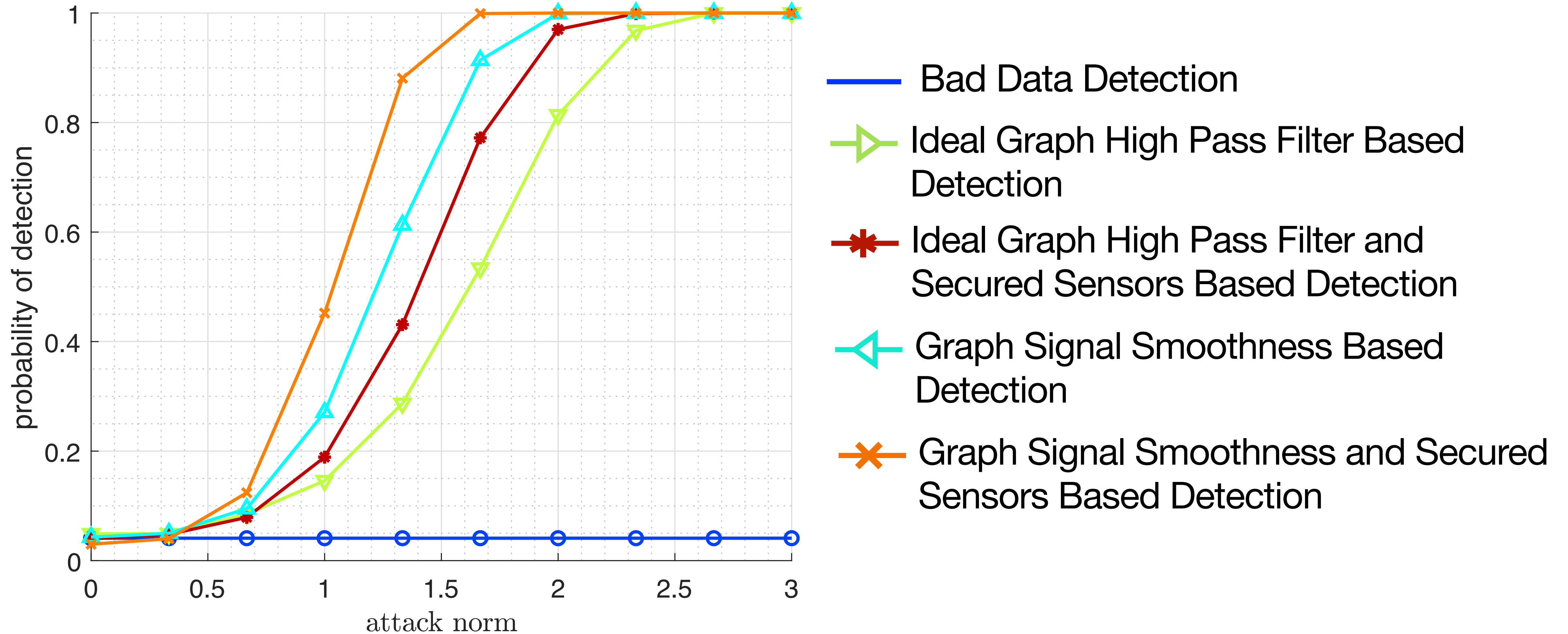
# Probability of Detection Verse Attack Norm



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# Probability of Detection Verse Attack Norm

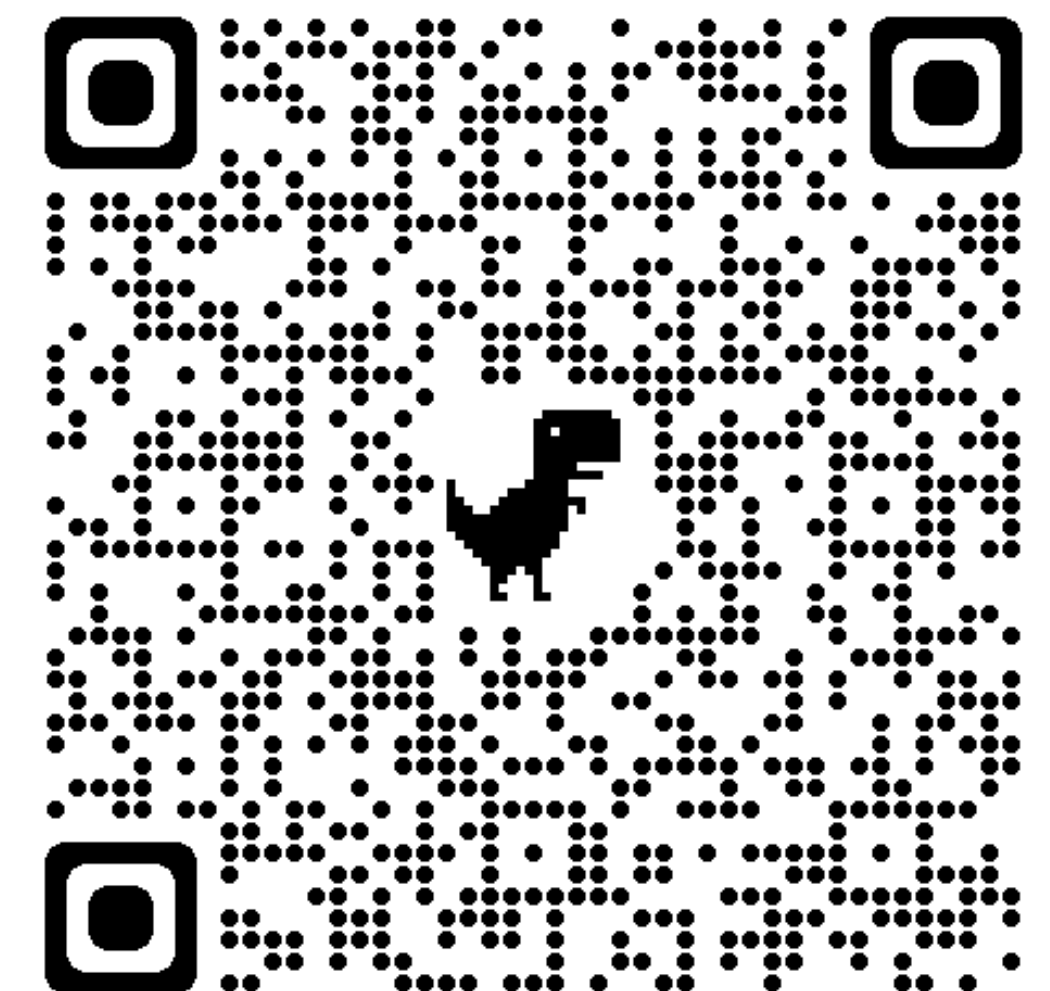


# Summary

- Introduced two regularization factors for power system state estimation under false data injection attacks
  1. Graph-based regularization on the system states
  2. Secured sensors-based regularization on the attack
- Provided a detection method against false data injection attacks
- Provided a modification of the detection method to distributed optimization



**Gal Morgenstern**  
[galmo@post.bgu.ac.il](mailto:galmo@post.bgu.ac.il)



# IEEE-57 Power System Test Case

